be reached after one transit time $T = d/u$, where T is the time required for a wave to propagate between the two ends of the transmission line. If the load is not matched, there will be a new reflected voltage wave v^- generated at $z = d$ and $t = T = d/u$. As the incident voltage v^+ continues to propagate in the positive z direction, the reflected voltage v^- propagates back toward the source. If the source end is matched, after one round trip $t = 2T = 2d/u$, there will be no further reflections, and the steady state will be reached after time $t = 2T$.

EXAMPLE 7.2 OPEN & SHORT CIRCUITED TL's

Consider the dc voltage of Figure 7.13a, which is switched onto a transmission line of characteristic impedance Z_o and terminated by a load resistance R_L. Discuss the process of multiple reflections for $R_L = 0$ (short circuit) and for $R_L = \infty$ (open-circuit termination), and determine the voltage and the current along the transmission line in both cases.

Solution

In Figure 7.13b it is clear that because the source resistance $R_G = Z_o$ and is, hence, matched to the transmission line, only half of the source voltage propagates down the line toward the load. If the transmission line is short circuited—that is, $R_L = 0$—the reflection coefficient at the load $\Gamma_T = -1$ so that $v^-/v^+ = -1$, and $i^-/i^+ = -\Gamma_T = 1$. In other words, the reflected voltage wave will cancel the incident wave and the reflected current wave will add, in phase, an equal value to the incident current wave as shown in Figure 7.13c. Because the source end is matched, no further reflections will occur at $z = 0$ with the arrival of the reflected voltage v^- and the reflected current i^- to the generator end, and the steady state is reached for $t \geqq 2T$. Similarly, if the transmission line is open circuited, $R_L = \infty$, $\Gamma_T = 1$; hence, $v^-/v^+ = 1$ and $i^-/i^+ = -1$. The reflected voltage wave will therefore add to the incident, whereas the reflected and incident current waves will cancel each other, as shown in Figure 7.13d.

Let us consider next the case in which the source and the load ends are not matched. In this case, reflections will occur at both ends, and the voltage and current waves will continue bouncing back and forth forever. It should be noted, however, that because reflection coefficients are less than unity (i.e., $\Gamma_G < 1$ and $\Gamma_T < 1$)*, each successive reflection will be reduced in magnitude, and after a few round trips, the changes in v^+ and v^- become negligible, and the steady state is approximately reached.

7.6 REFLECTION DIAGRAM

For the case of resistive termination of transmission lines, the development of the multiple reflections and the consequent build-up of the voltage and current from their initial to final (steady-state) values can be easily understood by using the reflection diagrams. Because these reflection diagrams are applicable for both voltage and cur-

*Γ_G and Γ_T are the reflection coefficients at the generator and the termination ends, respectively.

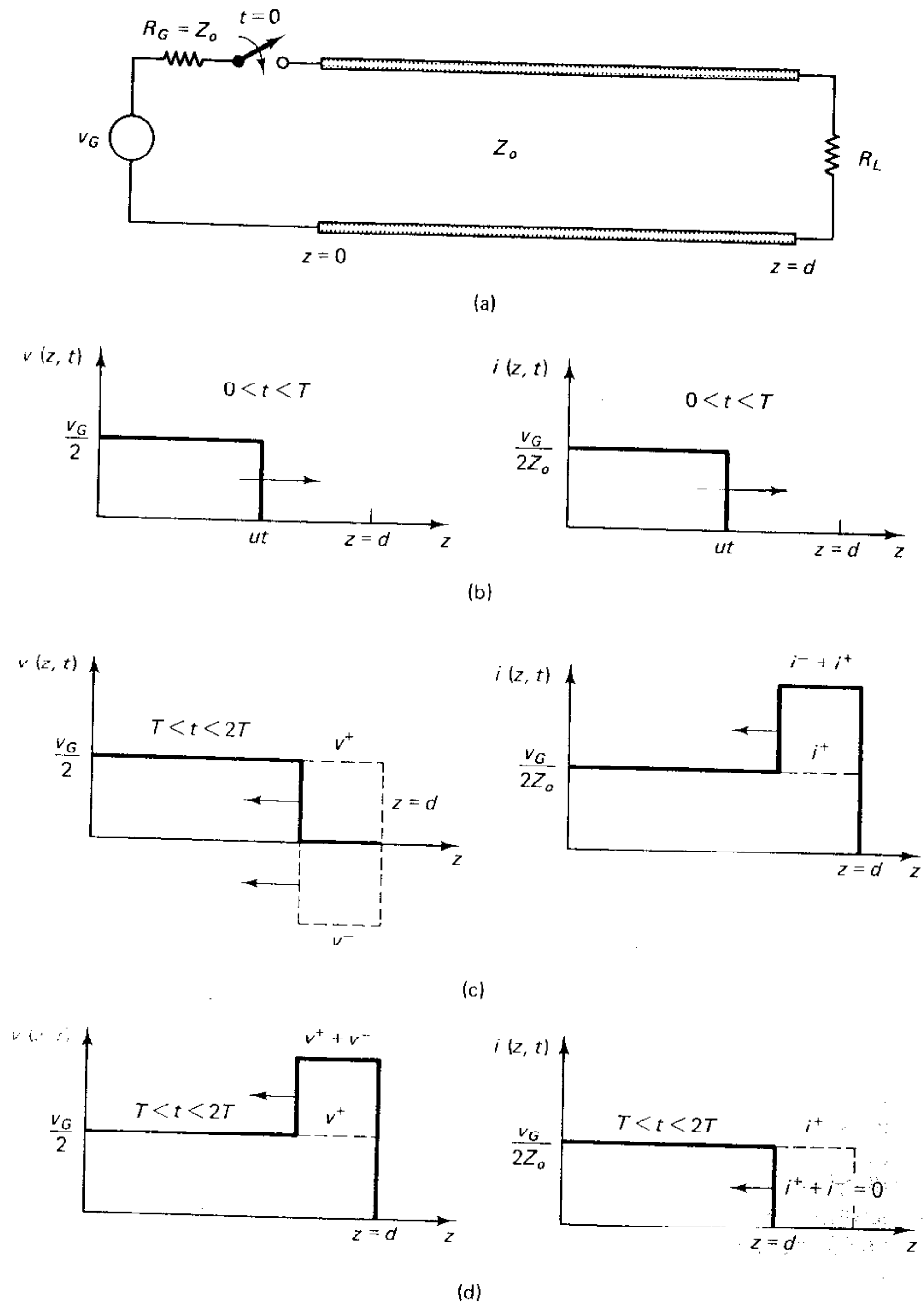


Figure 7.13a Transient analysis of a section of a transmission line terminated by a resistive load R_L. (b) Voltage and current distributions along a short-circuited transmission line for time $0 < t < T$. (c) Voltage and current distributions along the short-circuited transmission line for time $T < t < 2T$. (d) Voltage and current distributions along the open-circuit terminated transmission line for $T < t < 2T$.

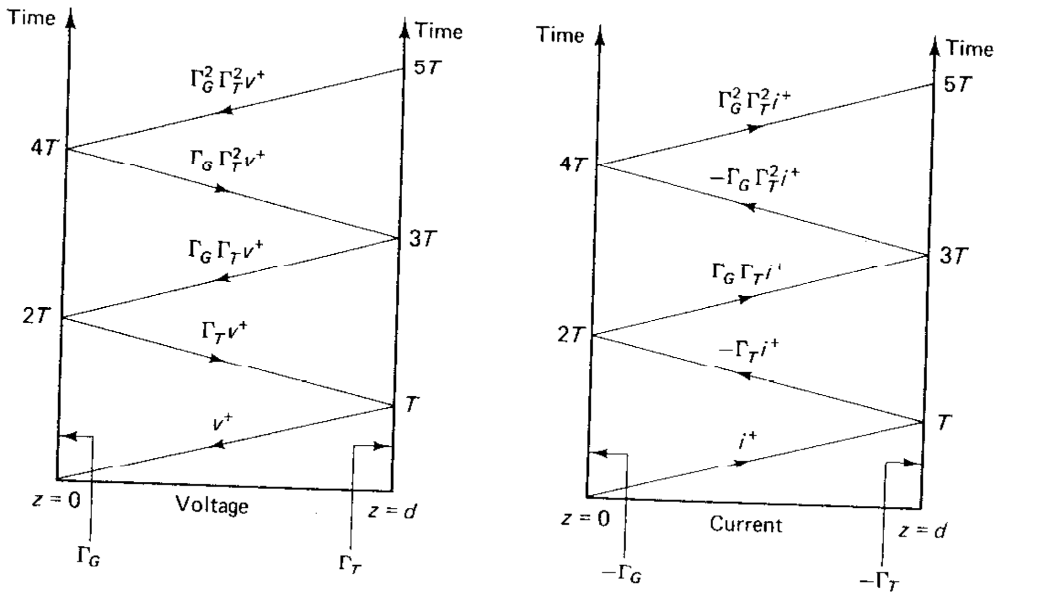


Figure 7.14 Voltage and current reflection diagram.

rent, it is usual to write "voltage" or "current," as the case may be, at the bottom of the reflection diagram. The abscissa of the reflection diagram denotes the length along the transmission line. In Figure 7.14, for example, the length extends from $z = 0$ to $z = d$. The reflection coefficients at the terminals have also been indicated at the appropriate positions along the abscissa as shown in Figure 7.14. The ordinate of the reflection diagram denotes the time required for the multiple reflections to develop along the transmission line. For example, the time required for an incident voltage or current to reach the termination at $z = d$ is $T = d/u$, where u is the velocity of propagation along the transmission line.

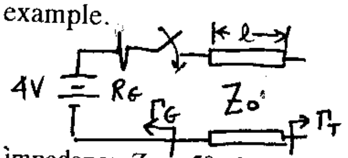
On the reflection diagram, the initial voltage along the transmission line (v^+) is marked at a point corresponding to the ordinate $t = 0$. In the time range $0 < t < T$, there is only v^+, which is the voltage wave traveling in the positive z direction and produced by the voltage generator at $z = 0$. At $z = d$ and $t = T$, an additional wave traveling in the negative z direction is generated. On the voltage reflection diagram, the reflected voltage is denoted by $\Gamma_T v^+$, whereas on the current reflection diagram, the reflected current is denoted by $-\Gamma_T i^+$. The negative sign in the current relations is simply included because the current reflection coefficient is the negative of the voltage reflection coefficient. The incremental voltages and currents corresponding to the successive reflections at both ends of the transmission line can thus be obtained in a similar manner.

There are two basic uses of the reflection diagram. These include the following:

1. Obtaining the voltage or current distribution along the transmission line at a given time.

2. Obtaining the voltage or current at any specified point on the transmission line as a function of time.

To illustrate these uses, let us consider the following example.



EXAMPLE 7.3

Consider a lossless transmission line of characteristic impedance $Z_o = 50$ ohms and extends from $z = 0$ to $z = 900$ m. The velocity of propagation along the transmission line is $u = 3 \times 10^8$ m/s. A battery of voltage 4 V and internal resistance $R_G = 150$ ohms is connected to the input terminal of the line ($z = 0$), whereas the output terminal is left open. Determine the following:

1. The voltage distribution along the line at time $t = 10 \times 10^{-6}$ s.
2. The time dependence of the voltage at $z = 600$ m, up to time $t = 12 \times 10^{-6}$ s.

Solution

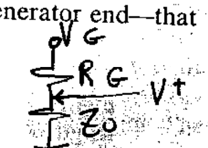
The voltage reflection diagram is shown in Figure 7.15a. The reflection coefficient at the termination Γ_T and the generator Γ_G ends are given by

$$\Gamma_G = \frac{150 - 50}{150 + 50} = \frac{1}{2}, \qquad \text{and} \qquad \Gamma_T = \frac{1 - Z_o/R_T}{1 + Z_o/R_T} = 1 = \left(\frac{R_T - Z_o}{R_T + Z_o}\right)$$

The time T required for a wave to travel the entire length of the transmission line is

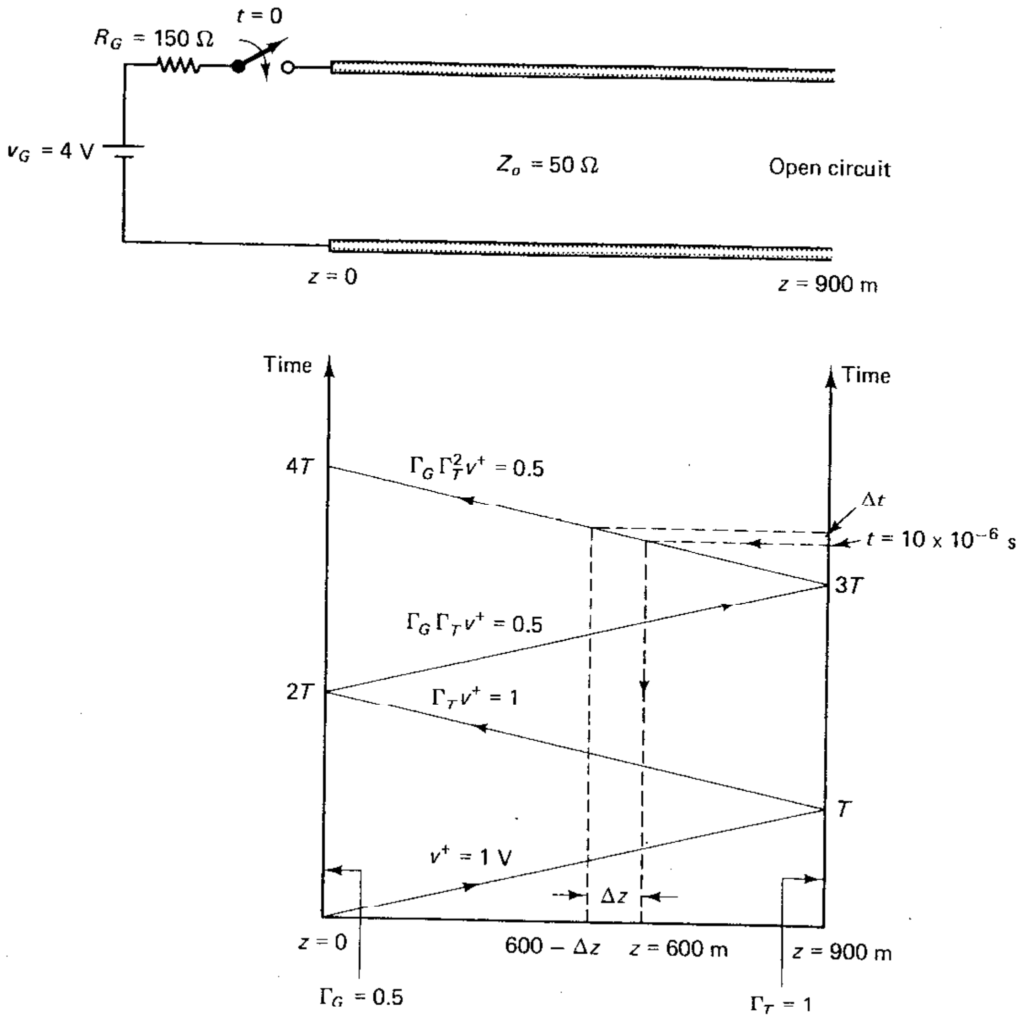
$$T = \frac{d}{u} = \frac{900 \text{ m}}{3 \times 10^8} = 3 \times 10^{-6} \text{ s}$$

The reflection diagram in Figure 7.15a is drawn for total time $t = 4T = 12 \times 10^{-6}$ s. The incident voltage v^+ is determined from the initial conditions at the generator end—that is,



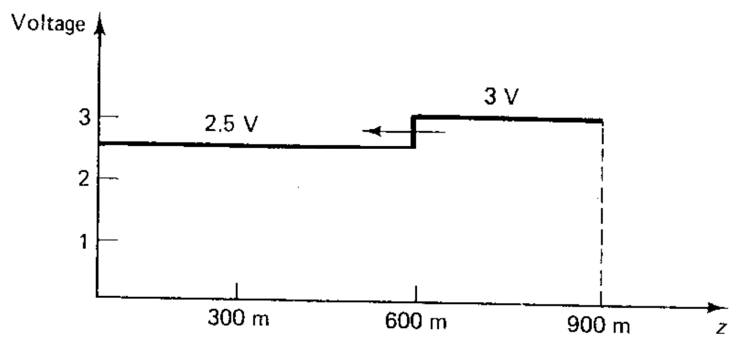
$$v^+ = \frac{v_G}{R_G + Z_o} Z_o = \frac{4 \times 50}{150 + 50} = 1 \text{ V}$$

1. To find the voltage distribution along the transmission line at $t = 10 \times 10^{-6}$ s, we draw a dashed line corresponding to the ordinate $t = 10 \times 10^{-6}$ s, and this intersects the locus straight line at an abscissa point corresponding to $z = 600$ m (see Figure 7.15a). This simply indicates that at $t = 10 \times 10^{-6}$ s there will be a voltage discontinuity located at $z = 600$ m. If the time is incremented by Δt—that is, from 10×10^{-6} to $(10 \times 10^{-6} + \Delta t)$, as shown in Figure 7.15a—it is clear that z changes from 600 m to $(600 - \Delta z)$, which indicates that the voltage discontinuity is traveling toward the generator end. The voltage distribution along the transmission line at $t = 10 \times 10^{-6}$ s can be obtained by adding the voltages that resulted from the multiple reflections (e.g., $\Gamma_T v^+$, $\Gamma_G \Gamma_T v^+$, etc.) with their appropriate signs to the incident voltage v^+. Of course, only the reflections that occurred at time t less than or equal to 10×10^{-6} s will be considered and counted in the addition process. From Figure 7.15a, we find that the total voltage along the transmission line from $z = 0$



(a)

Figure 7.15a Voltage reflection diagram for an open-ended transmission line of length 900 m.



(b)

Figure 7.15b Voltage distribution at $t = 10 \times 10^{-6}$ s.

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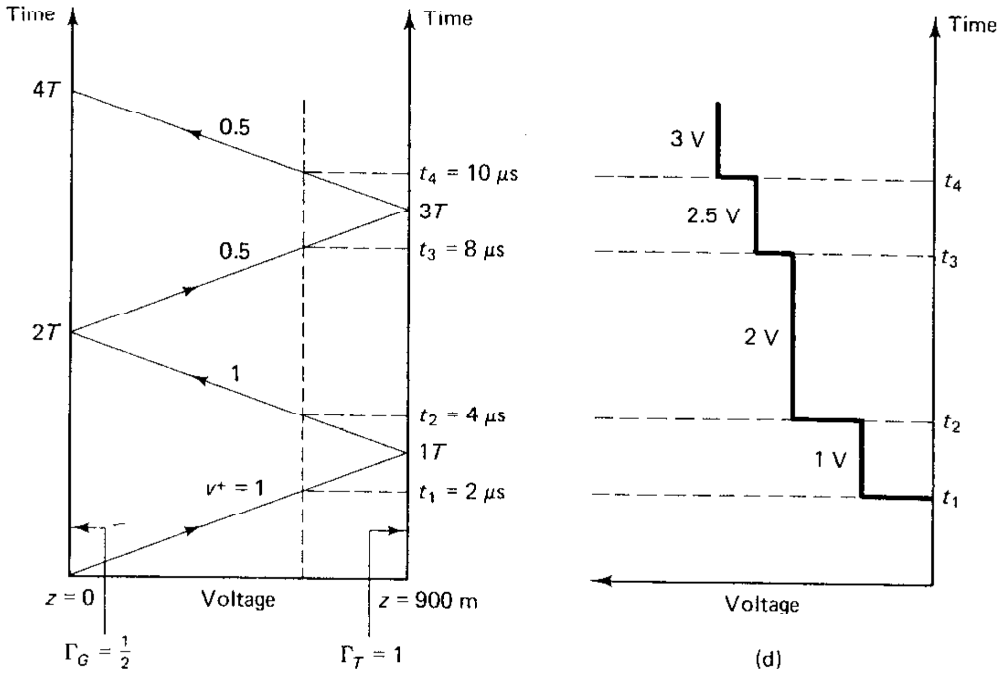


(c)

Figure 7.15 (c) Reflection diagram with specific emphasis on the voltage distribution at $z = 600$ m. (d) Voltage distribution at $z = 600$ m as a function of time.

to $z = 600$ m is 2.5 V and that from $z = 600$ m to $z = 900$ m is 3 V. Thus, we obtain the voltage distribution shown in Figure 7.15b.

2. To find the time dependence of the voltage at $z = 600$ m from an initial time $t = 0$ up to a time $t = 12 \times 10^{-6}$ s, we draw a dashed line corresponding to the abscissa point $z = 600$ m as shown in Figure 7.15c. This intersects the locus straight lines representing the multiple reflections at $t = t_1, t_2, t_3$, and t_4, where for this example $t_1 = 2 \times 10^{-6}$ s, $t_2 = 4 \times 10^{-6}$ s, $t_3 = 8 \times 10^{-6}$ s, and $t_4 = 10 \times 10^{-6}$ s. The development of the voltage wave form as a function of time is shown in Figure 7.15d. The voltage at $z = 600$ m is zero from $t = 0$ to 2×10^{-6} s, when the voltage changes to 1 V with the arrival of the incident voltage. At each of the subsequent times t_2, t_3, and t_4 indicated on Figure 7.15d, the voltage at $z = 600$ m changes by an amount indicated on the appropriate locus straight line representing the multiple reflection as shown in Figure 7.15c. In this manner we obtain the voltage at $z = 600$ m as a function of time up to $t = 12 \times 10^{-6}$ s.

7.7 TANDEM CONNECTION OF TRANSMISSION LINES

In addition to discontinuities at the generator and the termination locations on the transmission line, reflections may occur at junctions between various transmission lines. For example, consider two transmission lines of characteristic impedances Z_{o1} and Z_{o2} connected as shown in Figure 7.16.

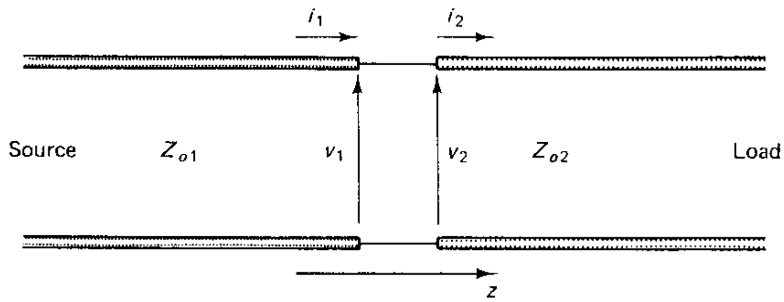


Figure 7.16 The junction of two transmission lines, source wave is incident from line 1.

The boundary condition at the junction of two transmission lines requires that the voltage and current be continuous across the junction—that is, $v_1 = v_2$ and $i_1 = i_2$. If the incident voltage wave is from line 1 (v_1^+), at the junction there will be a reflected wave v_1^- in line 1 and a transmitted wave v_2^+ in line 2. In this case, $v_1 = v_1^+ + v_1^-$, whereas $v_2 = v_2^+$. The voltage boundary condition is then satisfied when

$$v_1^+ + v_1^- = v_2^+$$

which may be rewritten as

$$1 + \frac{v_1^-}{v_1^+} = \frac{v_2^+}{v_1^+} \tag{7.29}$$

The ratio v_1^-/v_1^+ has been previously defined as the reflection coefficient Γ and will be denoted here as Γ_{11} to emphasize the fact that the reflection occurs at the end of line 1. Similarly, we define a transmission coefficient τ_{12} as

$$\tau_{12} = \frac{v_2^+}{v_1^+} \tag{7.30}$$

This coefficient gives the fraction of incident wave from line 1 that is transmitted through to the second line, hence, the coefficient τ_{12}. Equation 7.29 then becomes

$$\tau_{12} = 1 + \Gamma_{11} \tag{7.31}$$

An expression for Γ_{11} may be obtained by following a procedure similar to that of section 7.4.

The total voltage and current at the junction are given by

$$v_T = v^+ + v^-$$

$$i_T = i^+ + i^- = \frac{v^+}{Z_{o1}} - \frac{v^-}{Z_{o1}}$$

Z_{o2} acts as termination to transmission line 1. Therefore, $v_T/i_T = Z_{o2}$, and substituting expressions for v_T and i_T we obtain,

$$Z_{o2} = Z_{o1}\frac{v^+ + v^-}{v^+ - v^-}$$

or

$$Z_{o2} = Z_{o1}\frac{1 + \Gamma_{11}}{1 - \Gamma_{11}}$$

The expression for Γ_{11} is hence

$$\Gamma_{11} = \frac{Z_{o2} - Z_{o1}}{Z_{o2} + Z_{o1}}$$

From equation 7.31 the transmission coefficient is given by

$$\tau_{12} = \frac{2Z_{o2}}{Z_{o1} + Z_{o2}}$$

Let us next consider the situation where the source is moved to line 2, as shown in Figure 7.17.

In this case there will be an incident and a reflected wave in line 2 (v_2^+ and v_2^-), and only a transmitted wave in line 1 (v_1^+). Thus, the voltage boundary condition gives

$$v_1^+ = v_2^+ + v_2^-$$

or

$$\frac{v_1^+}{v_2^+} = 1 + \frac{v_2^-}{v_2^+} \tag{7.32}$$

If we define $v_2^-/v_2^+ = \Gamma_{22}$, the reflection coefficient in line 2, and $v_1^+/v_2^+ = \tau_{21}$, the transmission coefficient from line 2 to line 1, then we can rewrite equation 7.32 in the form

$$\tau_{21} = 1 + \Gamma_{22} \tag{7.33}$$

where

$$\Gamma_{22} = \frac{Z_{o1} - Z_{o2}}{Z_{o1} + Z_{o2}}$$

and from equation 7.33,

$$\tau_{21} = \frac{2Z_{o1}}{Z_{o1} + Z_{o2}}$$

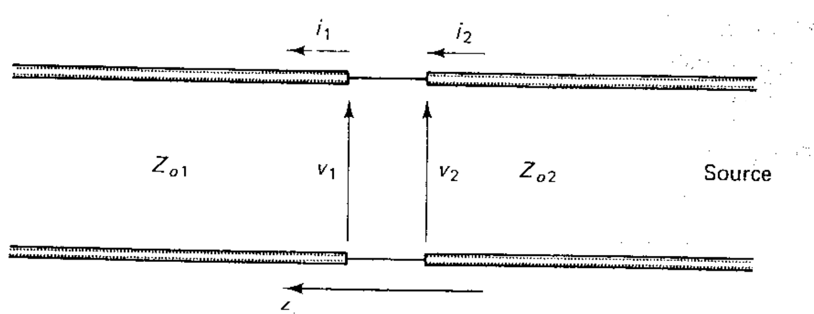


Figure 7.17 The junction of two transmission lines; source wave is incident from line 2.

The reflection diagram is a very useful and frequently used tool in determining the voltage and current at any point in a cascaded transmission-line system. To illustrate the development of the multiple reflections in this case, let us consider the following example.

EXAMPLE 7.4

Consider the tandem transmission line system shown in Figure 7.18. All the system parameters are shown in Figure 7.18. If the velocity of propagation in both transmission lines is the same—that is, $u_1 = u_2 = 3 \times 10^8$ m/s—determine the voltage distribution on the tandem system at time $t = 7.5 \times 10^{-8}$ s.

Solution

First we determine the reflection and transmission coefficients at the various junctions and terminations along the transmission line:

$$\Gamma_G = \frac{R_G - Z_{o1}}{R_G + Z_{o1}} = 0, \qquad \text{because } R_G = Z_{o1}$$

$$\Gamma_T = \frac{R_T - Z_{o2}}{R_T + Z_{o2}} = \frac{\frac{1}{3}Z_{o2} - Z_{o2}}{\frac{1}{3}Z_{o2} + Z_{o2}} = -\frac{1}{2}$$

$$\Gamma_{11} = \frac{Z_{o2} - Z_{o1}}{Z_{o2} + Z_{o1}} = -\frac{1}{2}$$

$$\tau_{12} = \frac{2Z_{o2}}{Z_{o1} + Z_{o2}} = \frac{1}{2}, \qquad \text{or} \qquad \tau_{12} = 1 + \Gamma_{11}$$

$$\Gamma_{22} = \frac{Z_{o1} - Z_{o2}}{Z_{o1} + Z_{o2}} = \frac{1}{2}$$

$$\tau_{21} = \frac{2Z_{o1}}{Z_{o1} + Z_{o2}} = \frac{3}{2}, \qquad \text{or} \qquad \tau_{21} = 1 + \Gamma_{22}$$

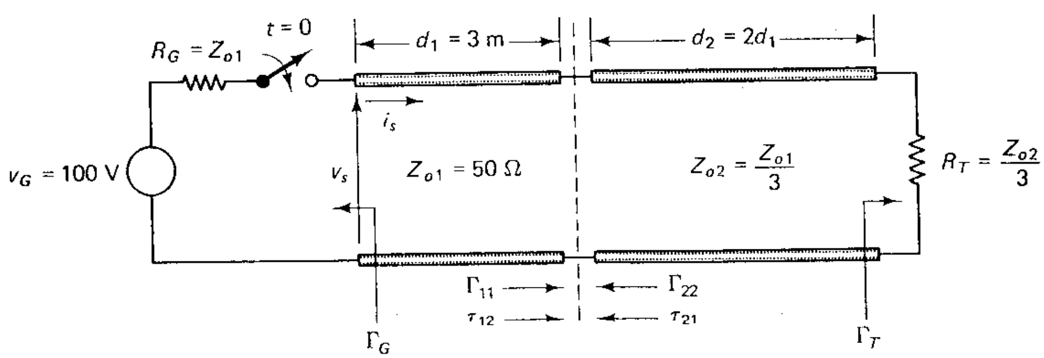
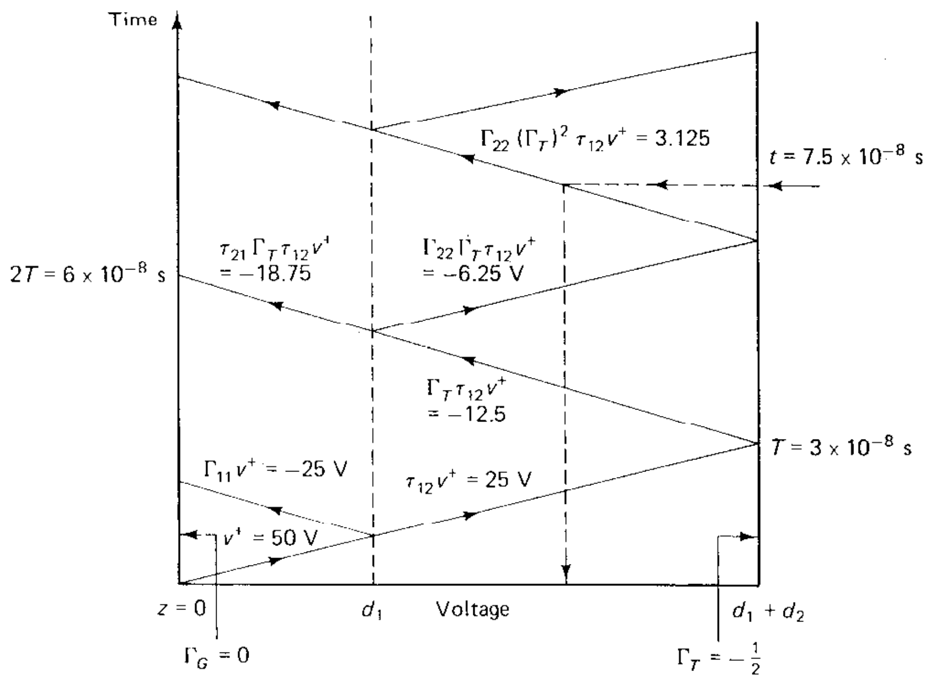
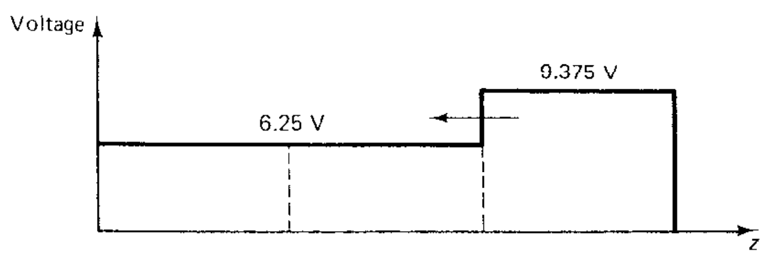


Figure 7.18 Tandem-line system.



(a)

Figure 7.19a Reflection diagram for a tandem connection of two transmission lines of the same velocity of propagation.



(b)

Figure 7.19b Voltage distribution at $t = 7.5 \times 10^{-8}$ s.

The wave initially launched on line 1 by the step generator is given by

$$v_s = v^+ = \frac{v_G}{R_G + Z_{o1}} Z_{o1} = \frac{v_G}{2} = 50 \text{ V}$$

The reflection diagram pertinent to this problem is shown in Figure 7.19a. The resulting voltage distribution along the line is shown in Figure 7.19b.

EXAMPLE 7.5

Two transmission lines 1 and 2 of characteristic impedances $Z_{o1} = 50\ \Omega$ and $Z_{o2} = 100\ \Omega$, respectively, are connected in tandem as shown in Figure 7.20. At $t = 0$, transmission line 1 is connected at one side to a battery of 3 V and internal resistance $R_G = 150\ \Omega$, and at the other side to the transmission line 2 through a series resistor $R_s = 100\ \Omega$. The transmission line 2 is terminated by a load resistance $R_L = 100\ \Omega$.

If the velocity of propagation along line 1 is $u_1 = 3 \times 10^8$ m/s, and the velocity of propagation along line 2 is $u_2 = 2 \times 10^8$ m/s, determine the following:

1. The voltage at the sending end for a period up to $t = 12 \times 10^{-6}$ s.
2. The voltage at the termination for a period up to $t = 9 \times 10^{-6}$ s.

Solution

We start the solution, like in all cases, by determining the reflection coefficients at all the discontinuities.

$$\Gamma_G \equiv \text{reflection coefficient between the source and transmission line 1}$$
$$= \frac{150 - 50}{150 + 50} = \frac{1}{2}$$

$$\Gamma_L \equiv \text{load reflection coefficient} = 0$$

$$\Gamma_{11} \equiv \text{reflection coefficient between lines 1 and 2}$$

$$\Gamma_{11} = \frac{v_1^-}{v_1^+}$$

To obtain a relation between v_1^- and v_1^+ when a series or any other combination of resistive discontinuities is present at the junction between the transmission lines, we use our basic equations that relate the current to the voltage at the junction. In transmission line 1,

$$v_T = v_1^+ + v_1^-$$

$$i_T = i_1^+ + i_1^- = \frac{1}{Z_{o1}}(v_1^+ - v_1^-)$$

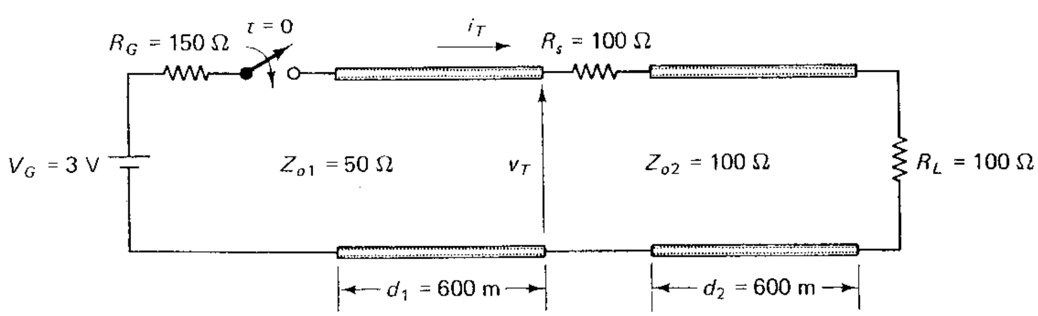


Figure 7.20 Tandem connection of two transmission lines through a series resistor R_s.

In Figure 7.20, however, v_T/i_T is simply $R_s + Z_{o2}$. Hence,

$$R_s + Z_{o2} = Z_{o1}\frac{1+\Gamma_{11}}{1-\Gamma_{11}}$$

The reflection coefficient Γ_{11} is then given by

$$\Gamma_{11} = \frac{(R_s + Z_{o2}) - Z_{o1}}{(R_s + Z_{o2}) + Z_{o1}} = 0.6$$

To determine $\tau_{12} = v_2^+/v_1^+$, we also need to relate v_2^+ to v_1^+:

$$v_1 = v_1^+ + v_1^- = v_1^+(1 + \Gamma_{11})$$

$$v_2^+ = \frac{v_1}{R_s + Z_{o2}}Z_{o2} = \frac{v_1^+(1+\Gamma_{11})Z_{o2}}{R_s + Z_{o2}}$$

$$\therefore \tau_{12} = \frac{v_2^+}{v_1^+} = \frac{(1+\Gamma_{11})Z_{o2}}{R_s + Z_{o2}} = 0.8$$

Looking from line 2 into line 1,

$$\Gamma_{22} = \frac{150 - 100}{100 + 150} = 0.2$$

Following a procedure similar to that we used in determining τ_{12}, we can determine τ_{21}:

$$\tau_{21} = \frac{v_1^+}{v_2^+} = \frac{(1+\Gamma_{22})50}{150} = 0.4$$

The incident voltage

$$v_1^+ = \frac{v_G Z_{o1}}{Z_{o1} + R_G} = \frac{3}{4}\text{ V}$$

The travel times T_1 and T_2 in transmission lines 1 and 2 are given respectively by

$$T_1 = \frac{600}{3 \times 10^8} = 2 \times 10^{-6}\text{ s}$$

$$T_2 = \frac{600}{2 \times 10^8} = 3 \times 10^{-6}\text{ s}$$

With the identification of all these reflection and transmission parameters, we are now ready to construct the reflection diagram shown in Figure 7.21. It should be noted that the slopes of the locus straight lines are different in the two different transmission lines. This is simply because the velocities of propagation are different in the two lines. The slope of the locus straight line is basically $1/u$; hence, larger slopes will be in the region of smaller velocities of propagation.

1. The required voltage at the sending end is obtained by constructing a vertical line at $z = 0$ (i.e., at the sending end) and adding the incremental contributions from the multiple reflections as the time elapses. For example, for time t between 0 and 4×10^{-6} s, we have at the sending end only the incident voltage of 3/4 V. At $t = 4 \times 10^{-6}$ s, the voltage reflected from the series discontinuity arrives at the sending end, and it simultaneously generates a reflected voltage wave as a result of

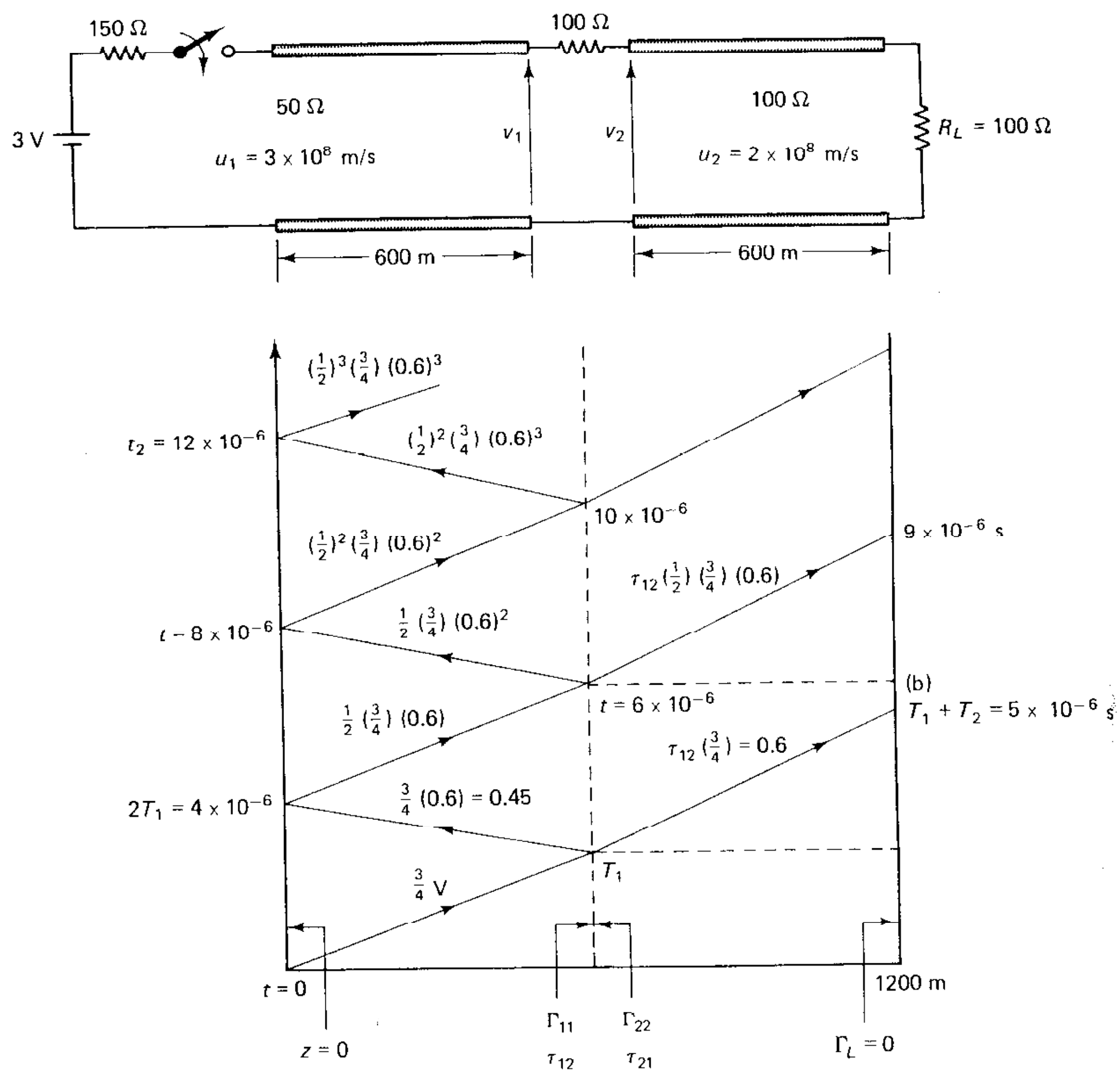


Figure 7.21 Bounce diagram for example 7.5. Tandem connection of two transmission lines of two different propagation velocities.

the mismatch between line 1 and the source. The total voltage at $t = 4 \times 10^{-6}$ is, hence,

$$\underset{V^+}{\frac{3}{4}} \quad + \quad \underset{\substack{\text{reflection from}\\\text{series}\\\text{discontinuity}}}{\Gamma_{11}\left(\frac{3}{4}\right)} \quad + \quad \underset{\substack{\text{reflection at the}\\\text{generator}}}{\Gamma_G\left[\Gamma_{11}\left(\frac{3}{4}\right)\right]}$$

The obtained result for the voltage at the sending end as a function of time is shown in Figure 7.22.

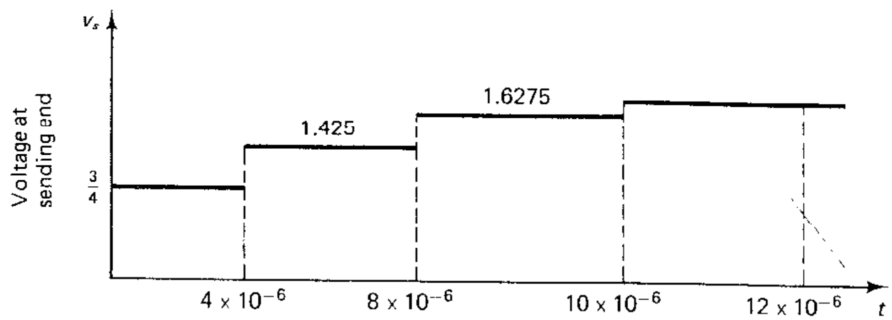


Figure 7.22 Voltage at the sending end as a function of time.

2. Similarly the voltage at the termination is obtained by adding the contributions from the multiple reflections at the load location. The obtained result is shown in Figure 7.23.

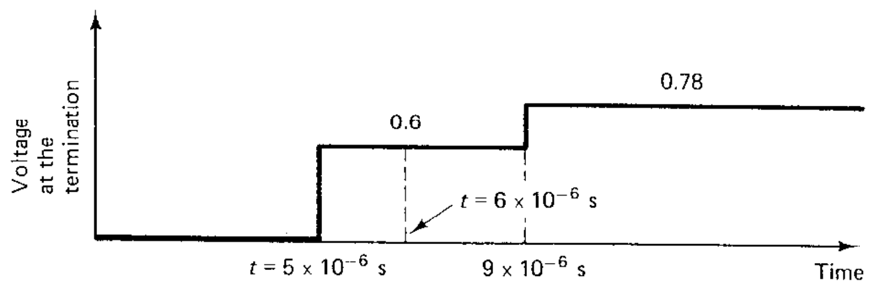
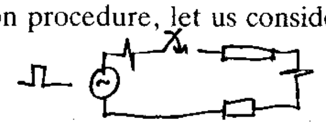


Figure 7.23 Voltage at the load as a function of time.

7.8 PULSE PROPAGATION ON TRANSMISSION LINES

Thus far we have considered only the propagation of dc transients on transmission lines. The same techniques can be used to study propagation of pulses on lines of finite lengths. To illustrate the solution procedure, let us consider the following example.

EXAMPLE 7.6



A transmission line of 50-ohm characteristic impedance and 600 m long is connected to a pulse generator that has an internal resistance of 150 ohms and produces a 40 volts, 1-μs pulse. The line is terminated by a load resistance of $R_T = 16.7$ ohms. If the velocity of propagation on the transmission line is 300 m/μs, determine the sending end voltage and current as a function of time.

Solution

The transmission line geometry is shown in Figure 7.24a. At $t = 0$, the incident voltage pulse is unaffected by the receiving end, and it begins to travel down the line as if the length

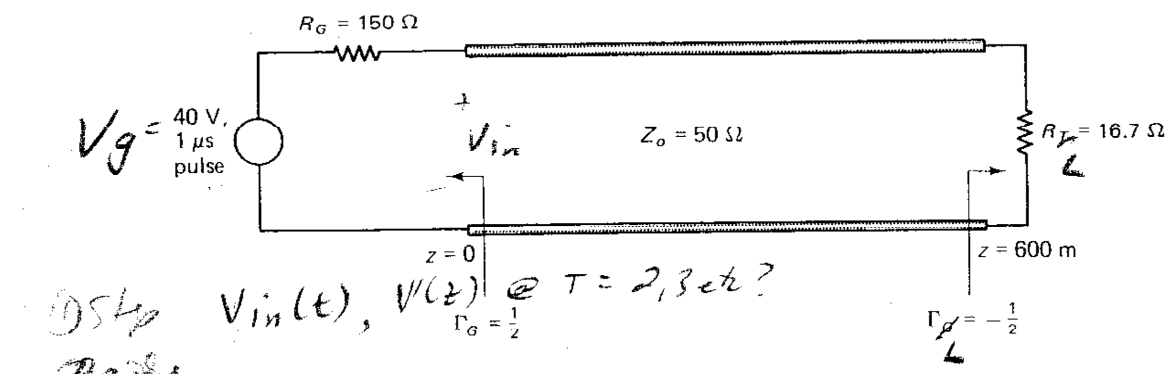


Figure 7.24a Pulse excitation of a 50 Ω transmission line of length d = 600 m.

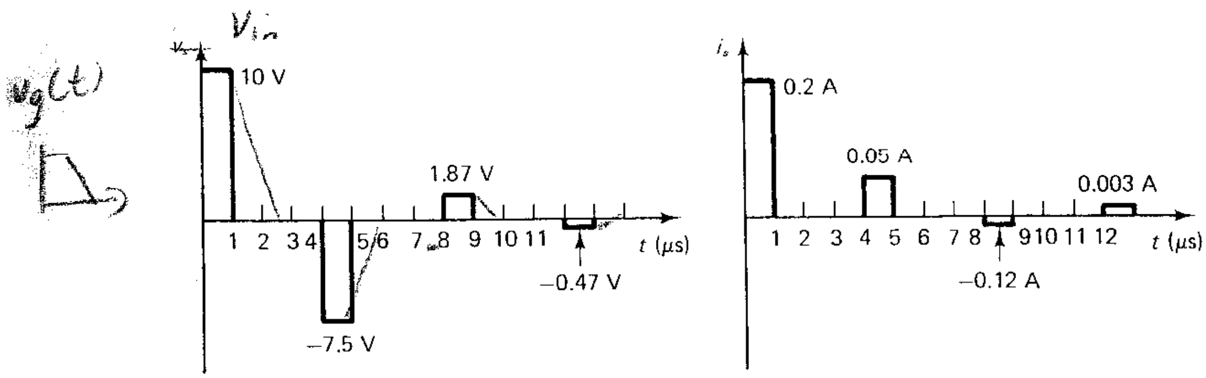


Figure 7.24b Sending-end voltage and current as a function of time.

of the line is infinitely long. The 40-V pulse is initially divided between the generator resistance R_G and the characteristic impedance Z_o of the line to give a sending-end pulse of

$$v_s = v^+ = \frac{40}{150 + 50} \times 50 = 10 \text{ V}$$

The 10-V pulse travels to the load in 2 μs and is partially absorbed and partially reflected, with the reflection $\Gamma_R = -1/2$. The reflected pulse, which is −5 V, travels back toward the generator in 2 μs and is partially reflected by the mismatched resistance R_G. The reflection coefficient at the generator is $\Gamma_G = 1/2$, which results in a reflected pulse of −2.5 V. Because the initially incident wave lasts only for 1 μs, its value does not contribute to the total voltage at the sending end for times larger than 1 μs—that is, $v^+ = 0$ for $t > 1$ μs. Therefore, the total voltage at $t = 2T = 4$ μs is basically the sum of the voltage reflected from the load (−5 V) and the reflected pulse at the generator end (−2.5 V). The second reflected pulse of −2.5 V results from the reflection of the load reflected voltage (−5 V) at the generator with $\Gamma_G = 1/2$. Therefore $v^{tot}(t = 2T) = -5 - 2.5 = -7.5$ V. From this point on, the process repeats itself with the pulse being multiplied by $\Gamma_T = -1/2$ at the termination and $\Gamma_G = 1/2$ at the generator, as shown in Figure 7.24b.

Similar procedure can be used for the current pulse with the initial current being $i^+ = 40/(150 + 50) = 0.2$ A. It should be noted, however, that the current reflection coefficient at the load is $i_T^-/i_T^+ = -\Gamma_T = 1/2$ and is $i_s^-/i_s^+ = -\Gamma_G = -1/2$ at the generator. The sending-end current is also shown in Figure 7.24b.

In summary, the process of pulse multiple reflections along transmission lines may be treated using the reflection diagram in a fashion similar to that used in the case in which we have step-voltage excitation. The only difference between the two cases is simply in the addition process of the contributions of the various reflections at a specific location along the transmission line to determine the voltage or current distribution along the line. In the step-voltage-excitation case, all the contributions from the multiple reflections that occurred at times previous or equal to the instant of interest are added in determining the final voltage or current distribution along the transmission line. In the pulse excitation case, however, this may or may not be the case depending on the duration of the pulse. In general, all contributions from the various multiple reflections should be considered, and only pulses (incident or reflected) that are still "on" at the desired instant of time will contribute to the value of the voltage or the current distribution at that time.

7.9 TIME-DOMAIN REFLECTOMETER

Evaluation of microwave components and devices may be routinely achieved by measuring their reflection and transmission properties as a function of frequency. For complete characterization, however, measurement over a broad frequency band is often desired. This broadband information can be obtained by sweeping the frequency in the desired range or by applying a short rise-time voltage pulse, or a step voltage, to the device under test. Based on a simple Fourier transformation of the applied pulse or step voltage, it can be shown that such a waveform contains a broad frequency band and that in the ideal case in which the applied voltage is a delta function, the frequency band extends from zero to infinity. Of course, it is impossible practically to generate a delta function voltage pulse with a zero rise time. This is why time-domain analysis does not provide frequency domain information over unlimited band. Instead, it provides frequency domain information in a band that is determined by the rise time of the incident pulse or step function voltage. The shorter the rise time, the broader the frequency band will be. This withstanding, it is clear that the desired broadband information can either be measured using swept-frequency techniques or by measuring the response of the system under test to an input short rise-time pulse- or a step-voltage excitation. In other words, the transient response of a microwave component or device together with a simple analysis procedure involving the Fourier transform may be used for a complete and broadband characterization of the device instead of the routine point-by-point or swept frequency domain measurements.

Another advantage of the time-domain pulse-type characterization of microwave systems is that it facilitates separating, in time, the responses from various discontinuities along the transmission-line system. In the frequency domain measurements where

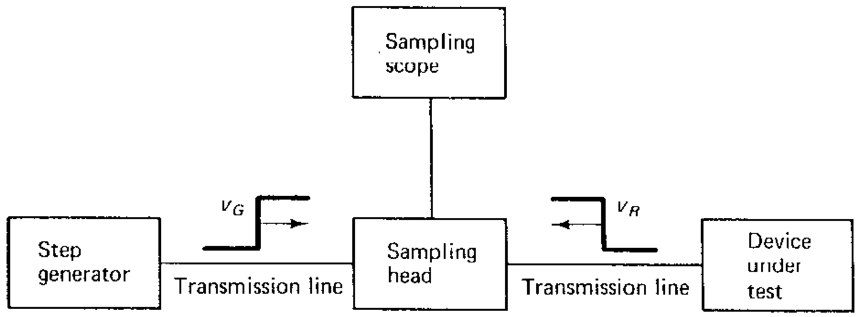


Figure 7.25 Basic components of the time-domain reflectometer.

sinusoidal voltages are often used, the composed reflection and transmission coefficients are measured, and it is quite difficult to decompose these measured values to the various components generated from the various discontinuities at different locations along the transmission line. In time-domain measurements, the contributions resulting from the various discontinuities along the transmission-line system are all separated in time and, hence, can be recognized separately. This is why time-domain or pulse measurement techniques have been used for many years to locate faults in cables and along telephone lines. Modern microwave automatic network analyzers provide gatting capabilities that facilitate isolation of reflections from various discontinuities and their subsequent analysis in the frequency domain.

A commercially available device that provides the capability of making pulse echo or time-domain measurements is known as the time domain reflectometer (TDR). A block diagram of this device is shown in Figure 7.25, where it can be seen that it consists of the following basic components:

1. A step-voltage generator that provides the step function input voltage to the transmission line.
2. A sampling head that includes a high-impedance probe to sample the voltage along the transmission line.
3. A sampling oscilloscope that is a broadband scope to display the probed voltage along the transmission line.
4. The device under test.

It should be noted that the sampling probe actually measures the total voltage $v_G + v_R$ along the transmission line. Therefore, to obtain the step-function response of the device under test, the incident step voltage should be subtracted from the total signal displayed on the scope.

The following are examples of some typical TDR displays and their interpretation.

7.9.1 Time-Domain Display of Resistive Terminations

Schematic diagram illustrating the operation principle of the TDR is shown in Figure 7.26. At $t = 0$, the step voltage is

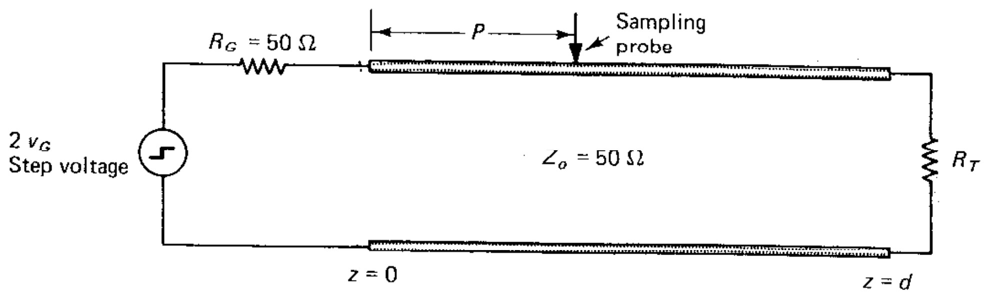


Figure 7.26 Schematic illustrating the function of the TDR when terminated by a resistance R_T.

$$v^+ = \frac{2v_G}{R_G + Z_o} Z_o = v_G \text{ V}$$

This incident voltage will be sampled by the high impedance probe at time $t = P/u$, where u is the velocity of propagation along the transmission line and P is the distance from probe to the generator. At $t = d/u$, this incident voltage v^+ will arrive at the termination. Assuming R_T is different from Z_o, there will be a reflected voltage v^- generated at $z = d$. The total voltage—that is, incident plus reflected—will be monitored by the sampling probe and displayed on the TDR scope. The time elapsed between the first monitoring of v^+ by the probe and the probing of the total voltage $v^+ + v^-$ is $2(d - p)/u$, which is basically the time required for a voltage wave to travel from the probe to the load and back to the probe.

At $t = 2d/u$, the reflected voltage will arrive at the generator, but because the generator's impedance R_G is equal to the characteristic impedance Z_o of the transmission line, no further reflections will occur, and steady state will be reached. As an example of the typical waveforms expected on the TDR, consider the case in which $R_T = 25\ \Omega$. In this case the reflection coefficient is given by,

$$\Gamma = \frac{R_T - Z_o}{R_T + Z_o} = \frac{25 - 50}{25 + 50} = -\frac{1}{3}$$

The reflected voltage v^- is then

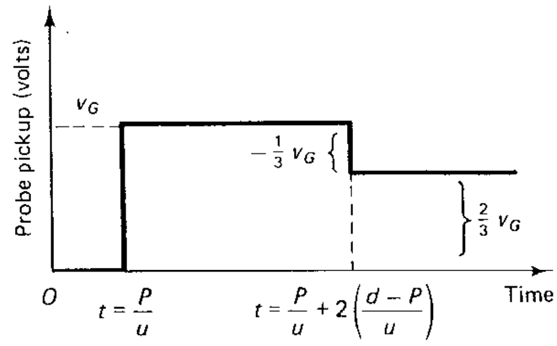
$$v^- = \Gamma v^+ = \Gamma v_G = -\frac{1}{3} v_G$$

The total voltage detected by the sampling probe is then $v^+ + v^- = 2/3 v_G$. A representative TDR waveform is shown in Figure 7.27a and a typical photograph of the TDR scope display is shown in Figure 7.27b.

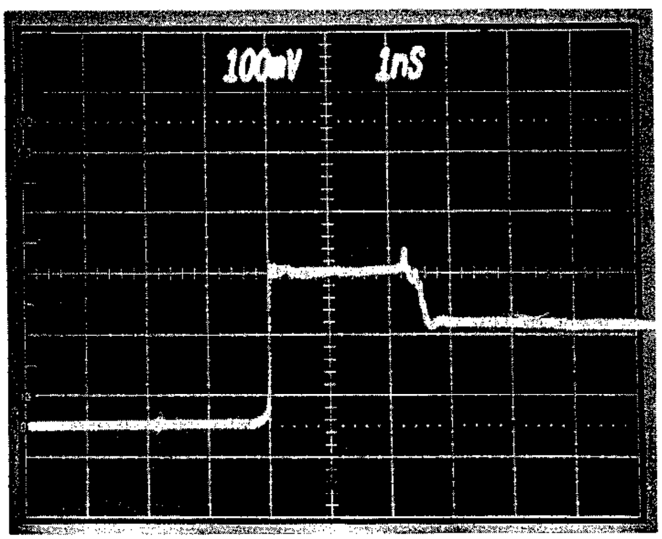
Other examples of possible TDR displays for a variety of resistive loads are shown in Figure 7.28.

7.9.2 Time-Domain Displays of Arbitrary Terminations

From the discussion in the previous section it is clear that resistive termination (R_T) of a lossless transmission line (Z_o real) results in a reflected voltage of the same shape as



(a) Schematic display of probe pickup voltage



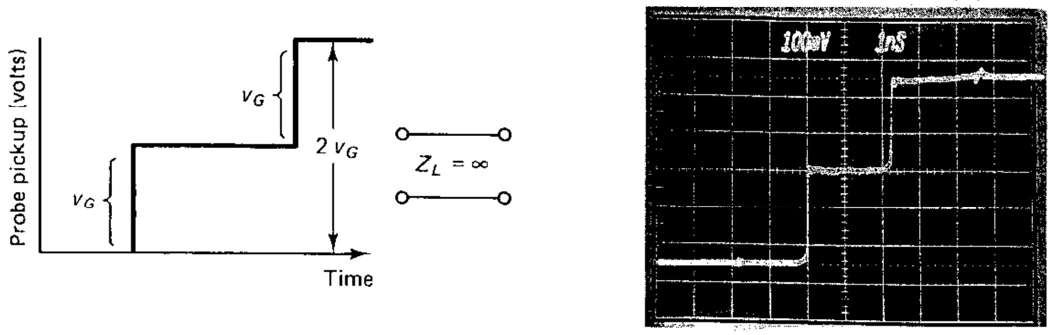
(b) Photograph of TDR scope

Figure 7.27 TDR display for $R_T = 25\ \Omega$.

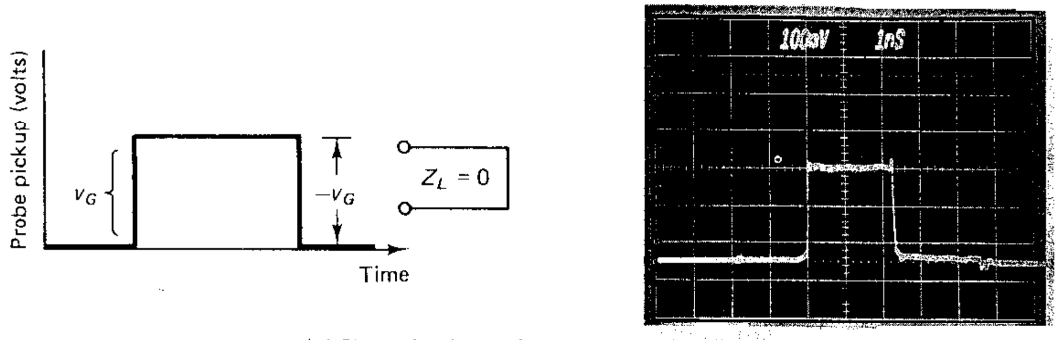
the applied one. The magnitude and polarity (positive or negative) of the reflected voltage, however, are determined by the specific value of R_T relative to Z_o.

In cases in which the transmission line is terminated by arbitrary impedances, the reflected voltage wave forms are generally different from the shape of the applied voltage. For example, consider the case in which we have a lossy capacitor terminating a lossless transmission line of characteristic impedance Z_o. A schematic illustrating the operation of the TDR with the lossy capacitor termination is shown in Figure 7.29.

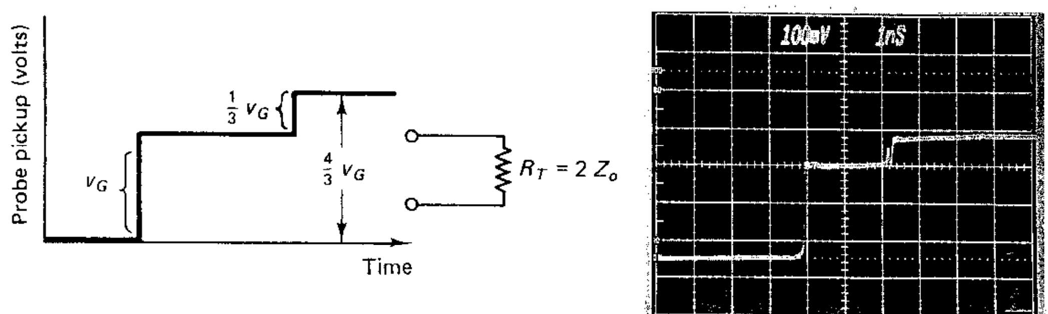
At $t = 0$, a step voltage $v^+ = 2v_G Z_o/R_G + Z_o = v_G$ will be applied to the transmission line. At $t = P/u$, the incident voltage will be sampled by the sampling probe and will also be displayed on the TDR scope. At $t = d/u$, the incident step will be applied to the capacitive termination, and a reflected voltage traveling back to the source will be generated at this point ($z = d$). To determine the reflected voltage, it



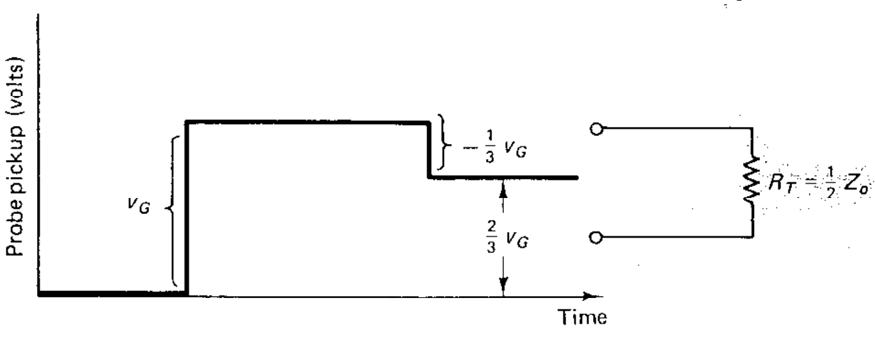
(a) Open-circuit termination $Z_L = \infty$ ($\Gamma = 1$)



(b) Short-circuit termination $Z_L = 0$ ($\Gamma = -1$)



(c) Termination resistance R_T equals double characteristic impedance Z_o ($\Gamma = 1/3$)



(d) Termination resistance $R_T = 1/2\ Z_o$ ($\Gamma = -1/3$).

Figure 7.28 TDR displays for various resistive terminations. (a) Open circuit termination $Z_L = \infty$ ($\Gamma = 1$), (b) Short circuit termination $Z_L = 0$ ($\Gamma = -1$), (c) Termination resistance $R_T = 2Z_o$ ($\Gamma = \frac{1}{3}$), and (d) Termination resistance $R_T = \frac{1}{2} Z_o$ ($\Gamma = -\frac{1}{3}$).

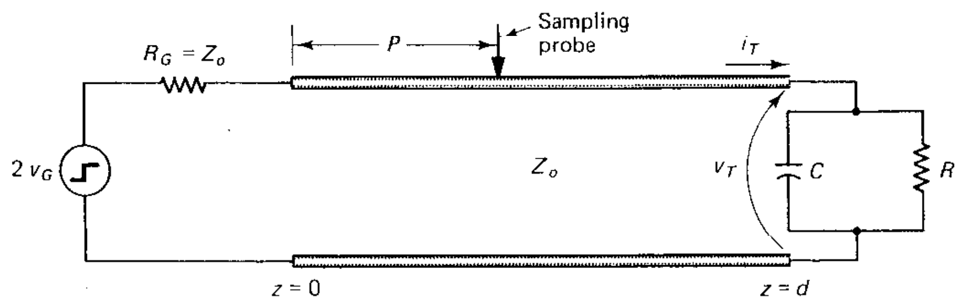


Figure 7.29 TDR terminated with a lossy capacitor, which is modeled as a parallel combination of R and C.

is necessary to use the arbitrary termination solution procedure described in section 7.4. It is shown that, at the termination, the incident voltage is related to the total voltage and current by

$$2v^+(d - ut) = v_T + Z_o i_T(t)$$

where t is measured from the time of arrival of v^+ at the termination. We will continue to develop our analysis based on this reference time t and will adjust it later to account for the reference $t = 0$ at the generator location. Substituting $v^+ = v_G$, and $i_T = C\,dv_T/dt + v_T/R$, which is the voltage current relation across the parallel load (i.e., $i_T = i_C + i_R$), we obtain

$$2v_G = \left(v_T + \frac{Z_o}{R}v_T\right) + Z_o C\frac{dv_T}{dt}$$

Once again the preceding equation is a first-order differential equation, and, for a dc input voltage (i.e., v_G = constant), the solution of this equation will take the form $v_T = v_t + v_f$ where v_t is the transient part of the response obtained by suppressing the source term and solving for the resulting homogeneous differential equation, and v_f is the forced part of the response which for dc input voltage is obtained by letting $dv_T/dt = 0$. It can be shown that

$$v_f = \frac{2v_G}{1 + \dfrac{Z_o}{R}}, \qquad v_t = Ae^{\frac{-t}{\tau}}$$

where τ in this case is the time constant given by $\tau = Z_o RC/(Z_o + R)$. The complete solution is, hence, given by

$$v_T = \frac{2v_G}{1 + \dfrac{Z_o}{R}} + Ae^{\frac{-t}{\tau}}$$

To determine the constant of integration A, we assume the condition that the charge on the capacitor was initially zero. Capacitors oppose sudden change in voltage; hence, $v_T = 0$ at $t = 0$.

Substituting this initial condition in the total voltage equation we obtain

$$A = \frac{-2v_G}{1 + \frac{Z_o}{R}}$$

The total voltage is, hence,

$$v_T = \frac{2v_G}{1 + \frac{Z_o}{R}}(1 - e^{\frac{-t}{\tau}}), \qquad \tau = \frac{Z_o R}{Z_o + R}C$$

To explain the resulting TDR display, let us rearrange the total voltage response equation in the form

$$v_T = \frac{2R\, v_G}{R + Z_o}(1 - e^{\frac{-t}{\tau}})$$

$$= v_G\left(1 + \frac{R - Z_o}{R + Z_o}\right)(1 - e^{\frac{-t}{\tau}})$$

The TDR response for this lossy capacitor termination is shown in Figure 7.30. From Figure 7.30, the following observations should be noted:

1. The sampling probe will first monitor $v_T = v_G$ traveling down the transmission line.
2. After time Δt (see Figure 7.30), which is the time required for the incident voltage to reach the termination and for the reflected voltage to travel back to the sampling probe—that is, $\Delta t = 2(d - P)/u$, the total voltage—that is, $v^+ + v^-$ along the transmission line—will be sampled by the probe and displayed on the scope.
3. With the arrival of the incident voltage at the termination and because the initial voltage on the capacitor is assumed to be zero, the overall termination will act initially as a short circuit. The reflected voltage wave v^- will be equal to

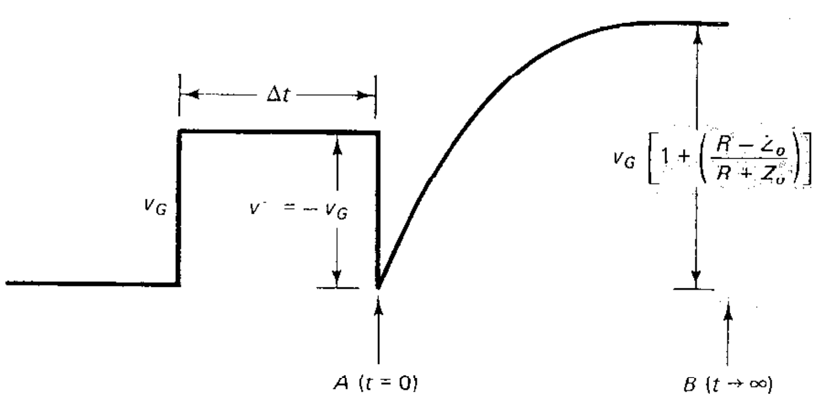


Figure 7.30 TDR display of a lossy capacitor termination.

$-v^+ = -v_G$, and the total voltage on the transmission line will, hence, be zero (point A) in Figure 7.30.

4. After a long time, the capacitor terminating the line will be fully charged and, hence, will act as an open circuit (i.e., current flow in the capacitor element = 0). The TDR termination will, hence, be basically the resistance R, and the reflected voltage in this case will be

$$v^- = v^+\Gamma = v_G\frac{R - Z_o}{R + Z_o}$$

The total voltage along the line as picked up by the sampling probe will, hence, be

$$v_T = v_G + v_G\frac{R - Z_o}{R + Z_o} = v_G\left(1 + \frac{R - Z_o}{R + Z_o}\right)$$

which is indicated by the point B in Figure 7.30.

5. The values of the total voltages at points A and B can also be obtained from the final expression of the total voltage simply by substituting the appropriate times $t = 0$ for point A and $t \rightarrow \infty$ for point B.

7.10 SINUSOIDAL STEADY-STATE ANALYSIS OF TRANSMISSION LINES

In the previous sections we developed the transmission-line equations, and for the case of lossless transmission lines we discussed the transients analysis of voltages and currents along these lines. We also indicated that such transient information may be used for locating faults along the transmission lines and telephone cables as well as in characterizing microwave components and devices over a broad frequency band with the aid of Fourier transformation.

Most practical applications of transmission lines, however, involve sinusoidal excitations. The 60-Hz power lines, broadcast stations, and other antenna operations are examples of sinusoidal excitations of transmission lines. In the following sections we will discuss the analysis and the various applications of transmission lines under sinusoidal excitation. In addition to the sinusoidal steady-state analysis of the voltage, current, and the power along transmission lines, we will discuss the use of sections of lossless lines to achieve impedance matching of microwave devices such as antennas. The use of slotted lines to measure the voltage and current characteristics along lossless lines and to measure unknown input impedances of microwave components will also be described.

7.10.1 Sinusoidal Steady-State Solution of Transmission-Line Equations

In section 7.2 we derived the transmission line equations without specifying the time variation of the voltage and the current along the line. We used these equations to study the step voltage and pulse excitation of lossless transmission lines. In the case of