Delay Performance of Threshold Policy for Dynamic Spectrum Access

Rong-Rong Chen*, and Xin Liu

Abstract

In this paper, we study the delay performance of secondary users (SU) under dynamic spectrum access. We design simple time-threshold policies for the SU to minimize the average delay while satisfying the collision probability constraint of the primary user (PU). Such policies perform closely to an optimized policy found by a Markov Decision Process (MDP) formulation, while facilitating analytical analysis of the delay and collision probability. For general busy and idle period distributions, we analyze the performance of the threshold policy through a one-dimensional Markov chain, and develop analytical expressions to approximate the delay and collision probability. The accuracy of the Markov chain analysis and the analytical approximations are examined under various busy and idle distributions. The capacity. impact of the busy and idle distributions on system performance are investigated. We find that while the idle distribution determines the time capacity, the busy distribution significantly affects the delay performance of the threshold policies.

Index Terms

Cognitive radio, dynamic spectrum access, delay, collision probability, threshold policy, Markov decision process.

I. Introduction

Cognitive Radio (CR) technology has great potential to alleviate spectrum scarcity in wireless communications. It allows secondary users (SUs) to opportunistically access spectrum licensed by primary users (PUs) while protecting PU activity. This new paradigm is typically referred to as *dynamic spectrum access* (DSA) [1]. In this paradigm, because the protection of PU is vital, a design imperative for an SU opportunistic access strategy is to minimize the SUs' effect on PU transmissions. For instance, the SU must guarantee that the collision probability of a PU packet

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is less than a threshold specified *a priori* by the PU. This type of constraint on the collision probability has been widely considered in the literature [2]– [5].

In this paper, we conduct analytical study of the delay performance of the SU under a collision constraint for the PU. This work is inspired from [6] in which the time capacity of the SU access is established under a collision constraint, assuming that the PU activity follows a general busy/idle time distribution. While we consider the same PU model as that of [6], in this work we adopt a different SU model to address a new problem in the design of transmission policies for DSA. The goal is to reduce the SU's access delay under PU protection, and to characterize the delay performance of the SU under these policies. In [6], the SU is assumed to be always backlogged in order to determine the time capacity. Thus, whenever a good spectrum opportunity appears, the SU can transmit. In comparison, in this work we assume that the SU's packet arrival follows a Bernoulli arrival process. Therefore, even when a good spectrum opportunity appears, the SU may not be able to transmit if it has an empty queue. Thus, due to the dynamics of the SU queue, the delay analysis developed here involves new techniques that are significantly different and more challenging than those of [6]. The threshold policies developed here, for minimizing delay, are also different from those found in [6], despite the similarity in the structure of these policies.

In this work, we first establish the SU's optimal access policy to minimize delay for general busy/idle time distributions using a Markov Decision Process (MDP) formulation. While the MDP provides an optimal policy, its calculation is cumbersome and provides little insight. This motivates us to develop a simple and more structured threshold policy that achieves near optimal performance. The main contribution of this work is that we analyze the performance of the threshold policies through a Markovian analysis, and develop closed-form approximations of the delay and collision probability for such policies under various PU busy/idle distributions. Numerical results confirm the accuracy of our approximations.

This work differs from other related work on the delay analysis for DSA networks in that we explicitly consider the collision constraint in the delay analysis, which is missing in other work such as [7]. Furthermore, our work considers general busy/idle time distributions, which also differs from that of [8] where the analysis is developed assuming exponentially distributed busy/idle time. We assume the PU activity to be unslotted, as opposed to [5], which assumes a slotted structure. Due to the technical challenges of the theoretical analysis, in this paper we limit ourselves to consider the case of a single SU accessing a PU channel, possibly shared by multiple PUs. In the analysis we also make the idealized assumption of perfect sensing and provide only numerical results for the imperfect sensing case. Extensions of the analysis to the more realistic case of multiple SUs and multiple PU bands are important directions for future research, but are out of the scope of this paper.

The remainder of the paper is summarized as follows. In Section II, we introduce the system model that characterizes the PU and SU activities. In Section III, we present the optimal MDP policy and the time-threshold policies. We analyze the performance of the threshold policies in Section IV through Markovian analysis, and derive closed-form analytical expressions to approximate the delay and collision probabilities of such policies in Section V. Numerical results are presented in Section VI. Finally, we conclude in Section VII.

II. System Model

In this section we describe our system model. We consider one spectrum band that is assigned to the PU, and one SU that opportunistically exploits the spectrum opportunities vacated by the PU under the protection requirement of the PU. While it is possible that there are multiple PUs sharing the spectrum band, we assume that the SU does not distinguish among different PUs, and can only access the channel when no PU is active. Thus, the SU treats all PUs collectively as one “aggregated” PU in designing the spectrum access schemes.

A. Primary User Model

We assume that the PU activities follow an alternating busy-idle pattern. Multiple PU packets, possibly with various lengths, are transmitted within a busy period. When all PU packets in the queue have been transmitted, the PU channel becomes idle. The PU channel remains idle until the arrival of the next PU packet, which is the start of the next busy-idle cycle. We denote the sojourn time of the PU idle state as I, its probability density function (PDF) as $f_I(\cdot)$, its cumulative distribution function (CDF) as $F_I(\cdot)$, its mean as $\mu_I = \int v f_I(v)dv$, and its second moment as $\nu_I = \int v^2 f_I(v)dv$. Similarly, B, $f_B(\cdot)$, $F_B(\cdot)$, μ_B, ν_B represent the sojourn time of the PU busy state, the pdf, the cdf, the mean, and the second moment, respectively. The percentage of time that the PU channel is idle is $\mu_I/(\mu_I + \mu_B)$, which is an upper-bound on the percentage of time that the SU can transmit on the PU channel. Note that while our results are applicable to arbitrary time-scales of the PU busy/idle time, the design of the DSA is simpler and it can achieve higher capacity if the time-scale of the PU busy/idle time is relatively large compared to that of a packet transmission time.

B. SU Model

We consider a packetized, time-slotted system for the SU. The SU has a fixed packet length that is no greater than that of the PU. Smaller values of the SU packet length provide more freedom for designing the SU access strategy. The arrival process of the SU is modeled as a Bernoulli process such that with probability p an SU packet arrives in a time slot and with probability $1-p$ there is no packet arrival. Each SU time slot consists of a short sensing period, followed by a transmission period, as shown in Fig. 1. The SU senses the channel during the sensing period. If the channel is sensed busy, then the SU does not transmit in the remainder of the time slot, and will sense the channel again in the sensing period of the next SU time slot. If the channel is sensed idle, then the SU has the option to either transmit, or not transmit, according to some transmission policy described in Section III. For example, the SU does not transmit in time slot 2 after sensing, either because its queue is empty or because its transmission policy decides so. If the SU transmits and the PU channel remains idle for the entire duration of the SU transmission period, then the transmission is successful. Otherwise, in the event that the PU returns in the middle of the SU transmission period, a packet collision occurs. This is illustrated in Fig. 1, where it shows that a collision occurs in time-slot k. For ease of presentation, in this paper we do not consider PU/SU packet re-transmission in the event of a collision, even though such modifications should be straightforward. We also assume an infinite buffer size at the SU and thus the packet dropping probability is not considered here.

C. PU Collision probability requirement

We denote p_c as the average packet collision probability "perceived" by the PU in the long-run, given by

$$p_c = \limsup_{K\to\infty} \frac{\sum_{k=0}^{K} N_c(k)}{\sum_{k=0}^{K} N_p(k)}, \tag{1}$$

where $N_c(k)$ and $N_p(k)$ are random variables representing the total number of collided and transmitted packets of the PU in the k-th busy-idle cycle, respectively. The PU has a packet collision probability requirement such that $p_c \leq \eta$, which is imposed by either the PU or the spectrum regulators and is known to the SU *a priori*. Under the assumption that the packet length of the SU is no greater than that of the PU, and that the sensing outcome of the SU is perfect, we note that *there is at most one PU packet collision within a busy-idle cycle, which may only occur at the beginning of a busy period when the PU returns after the SU has already sensed the channel to be idle and started a transmission.* During the next SU time slot, the SU will sense the channel to be busy and refrain from transmission, thus avoiding additional collisions with the PU packets. The analysis developed in this paper is under such assumption. When the sensing outcome is not error-free, the SU can possibly miss detect the PU activities, thus causing multiple PU packets collisions within a busy period. We will address the latter scenario through simulation in Section VI.

III. Transmission Policies for Minimizing Delay

In this section, we study transmission policies to minimize the average queueing delay of the SU under the collision constraint $p_c \leq \eta$. We assume that the SU has knowledge of η, and the PU busy/idle time distribution, i.e., $f_I(\cdot)$, and $f_B(\cdot)$. An option to obtain an optimal policy is to use the powerful tool of Markov Decision Process. In particular, the state space is two-dimensional: time and queue length; and the action is either to transmit or not to transmit. Through an MDP formulation, we compute the optimal transmission policy that minimizes the average cost in an infinite horizon. The cost considered here has two components: the delay cost and the collision cost. The collision cost can then be adjusted numerically to meet the collision probability constraint. Due to space limitation, we refer the readers to [9] for a detailed description of the MDP formulation.

Using the MDP formulation, we can numerically calculate an optimal MDP policy to minimize the delay while satisfying the collision probability constraint. However, the calculation is cumbersome and with numerical errors (e.g., while both the number of packets in the queue and the lengths of the busy/idle periods can go to infinity, we have to truncate them in numerical calculation). In addition, the MDP policy provides little insight on the relationship between delay and other system parameters. Therefore, we are motivated to look for a more structured policy.

It is shown in [6] that, for the case of backlogged traffic, the optimal SU transmission policy that achieves the time capacity of a PU channel is a time-threshold policy, i.e, the SU should transmit only when the elapsed time since the channel has been idle, denoted by t, is below a threshold Γ^*. The intuition behind this is that the SU should transmit only when the probability of a collision with the PU is small. For most idle distributions considered, [6] shows that conditioned upon t, the probability of a collision due to an SU transmission at time t is an increasing function of t. This naturally yields a time-threshold policy so that the SU will transmit only when t is below a threshold.

We can easily adapt the time-threshold policy of [6] here to the case when the arrival process of the SU packets is dynamic. The time-threshold policy with threshold Γ is defined such that the SU will transmit only when the following three conditions are met. (i) the channel is sensed idle. (ii) $t < \Gamma$ (note that this Γ is in general different from the threshold Γ^* that maximizes capacity). (iii) the SU queue length M is greater than zero. The time-threshold Γ should be adjusted to satisfy the PU collision probability constraint. As shown in Sections IV and V, the simplicity of time-threshold policies facilitates theoretical analysis of the delay and collision probability. Furthermore, to examine the effectiveness of the time-threshold policy, we compare it with the optimized policy found by the MDP formulation. We find that the time-threshold policy performs very closely to the optimized the MDP policy despite its simplicity. This reveals that the elapsed time t is the major factor that affects the delay and collision probability of a transmission policy, hence justifies the usage of the time-threshold policy considered here.

IV. Markov Chain Analysis for Threshold Policy

In this section, we develop Markovian analysis to analyze the delay and collision probability of threshold policies. Since the SU is not synchronized with the PU, the SU can only estimate the lengths of the PU busy and idle period through periodic sensing. In the remainder of this section, the lengths of the busy and idle periods refer to the lengths of these observed by the SU, which are integer multiple of the length of an SU time slot. For instance, if the SU detects the PU channel to be busy for B consecutive SU slots (sensing is done only once within each SU slot), then the length of the busy period (observed by the SU) is B. The length of the idle period I is defined similarly. Based on this definition, here B and I both take positive integer values. To simplify analysis, we assume perfect sensing, i.e., the length of the sensing period is zero, and the sensing outcome is error-free.

A. Formation of a one-dimensional Markov Chain

Assume that the k-th PU busy-idle cycle observed by the SU consists of B_k busy slots and I_k idle slots. To model the dynamics of the number of SU packets in the system, we define

$X_0^{(k)} = \{$Number of SU packets at the beginning of the k-th busy period$\}$

$Y_0^{(k)} = \{$Number of SU packets at the beginning of the k-th idle period$\}$

$X_i^{(k)} = \{$Number of SU packets at the end of the i-th slot of the k-th busy period, $1 \leq i \leq B_k\}$

$Y_j^{(k)} = \{$Number of SU packets at the end of the j-th slot of the k-th idle period, $1 \leq j \leq I_k\}$

Clearly, we have $Y_{I_k}^{(k)} = X_0^{(k+1)}$ because the number of SU packets at the end of the k-th idle period equals the number of the SU packets at the beginning of the $(k+1)$-th busy period. Similarly, $Y_0^{(k)} = X_{B_k}^{(k)}$ also holds. The dynamics of the number of the SU packets is characterized by the sequence of random variables

$$X_0^{(1)}, X_1^{(1)}, \cdots, X_{B_1}^{(1)} = Y_0^{(1)}, Y_1^{(1)}, Y_2^{(1)}, \cdots, Y_{I_1-1}^{(1)}, Y_{I_1}^{(1)} = X_0^{(2)}, X_1^{(2)}, \cdots, X_{B_2}^{(2)}, Y_1^{(2)}, Y_2^{(2)}, \cdots$$

The above sequence of random variables does not form a Markov chain, because the number of SU packets at time n depends on how long the PU channel has been in a busy/idle state. One can overcome this problem by tracking not only the number of SU packets, but also the elapsed time from the last busy-idle change. This approach, however, results in a Markov chain with a large two-dimensional state space and thus yields prohibitive complexity.

Our approach is based on the observation that the sequence of random variables $\{X_0^{(k)}, k = 1, 2, \cdots\}$ at the beginning of each busy-idle cycle forms a Markov chain. In other words, we can treat each busy-idle cycle as a single step in a discrete-time Markov chain and average over the lengths of the busy/idle periods B and I to compute the one-step transition probability matrix $P^{(1)}$. Specifically, we define

$$P_{i,j}^{(1)} = P(X_0^{(k+1)} = j | X_0^{(k)} = i) = P(Y_{I_k}^{(k)} = j | X_0^{(k)} = i). \tag{2}$$

Next, we consider a typical busy-idle cycle and drop the index k to write

$$\begin{aligned} P^{(1)}_{i,j} &= P(Y_I = j|X_0 = i) = \sum_{v=1}^{\infty} P(Y_v = j|X_0 = i)P(I = v) \\ &= \sum_{v=1}^{\infty} \left[\sum_{u=1}^{\infty} P(Y_v = j|X_0 = i, B = u)P(B = u) \right] P(I = v). \end{aligned} \tag{3}$$

Note that the term $P(Y_v = j|X_0 = i, B = u)$ in (3) depends on the threshold Γ of the transmission policy. Details for computing $P^{(1)}$ can be found in Appendix A. Assume that the steady-state distribution of the Markov chain is $\pi = (\pi_0, \pi_1, \cdots, \pi_n, \cdots)$ where π_n denotes the probability that there are n SU packets at the beginning of a busy period. We can find π by solving the equation $\pi P^{(1)} = \pi$. The average number of SU packets at the beginning of a busy-idle cycle thus equals $E(X_0) = \sum\limits_{i=0}^{\infty} i\pi_i$.

B. Calculation of Average Delay

The average number of the SU packets in the system is given by

$$N = \frac{E(X_0 + \cdots + X_{B-1} + Y_0 + \cdots + Y_{I-1})}{\mu_B + \mu_I}. \tag{4}$$

Following Little's formula [10], the average delay is $W = N/p - 1$. Here W excludes the time slot that the SU packet is transmitted.

In Appendix B, we show that W can be computed as

$$\begin{aligned} W = \frac{E(X_0)}{p} - 1 + \frac{1}{(\mu_B + \mu_I)p} \Bigg\{ & p\Big(\frac{\nu_B - \mu_B}{2} + \mu_B\mu_I\Big) + \frac{p-1}{2} \sum_{v=1}^{\Gamma} P(I = v)(v-1)v \\ & + \sum_{v=\Gamma+1}^{\infty} P(I = v)\bigg((p-1)\frac{\Gamma(1+\Gamma)}{2} + (v - \Gamma - 1)(p-1)\Gamma + \frac{p(v-\Gamma-1)(v-\Gamma)}{2}\bigg) \\ & + \sum_{v=1}^{\Gamma} P(I = v) \sum_{n=0}^{v-2} P(Y_n = 0)(v - 1 - n) + \sum_{v=\Gamma+1}^{\infty} P(I = v) \sum_{n=0}^{\Gamma} P(Y_n = 0)(v - 1 - n) \Bigg\}, \end{aligned} \tag{5}$$

where $P(Y_n = 0) = \sum\limits_{i=0}^{\infty} \pi_i P(Y_n = 0|X_0 = i), n = 1, \cdots, \Gamma$, has been computed in (3).

C. Calculation of Collision Probability

Given the threshold Γ, an SU transmits during the t-th slot of an idle period if $t \leq \Gamma$ and the number of packets at the end of the $(t-1)$-th idle slot (at the beginning of the t-th idle slot) is greater than zero, i.e., $Y_{t-1} > 0$. This transmission will result in a collision if the PU returns during the t-th slot, i.e., $I = t$. Hence, we can compute p_c as

$$p_c = \frac{1}{E(N_p)} \sum_{t=1}^{\Gamma} P(I=t)P(Y_{t-1} > 0) = \frac{1}{E(N_p)} \sum_{t=1}^{\Gamma} P(I=t)[1 - P(Y_{t-1} = 0)], \tag{6}$$

where $E(N_p)$ is the average number of PU packets per busy period. Here we have used the fact thatat most one packet collision occurs for each busy-idle cycle assuming perfect sensing.

V. Analytical approximations for the threshold policy

In this section, we derive analytical approximations for the delay and collision probability under the threshold policy. We first adopt the steady-state analysis to obtain a key approximation for $E(Y_0)$, the average number of packets at the beginning of an idle period. Based on this approximation, we further analyze the delay and collision performance.

A. Approximation of $E(Y_0)$

Let $n_1 = E(Y_0)$ and $n_0 = E(X_0) = E(Y_I)$. We will first derive an analytical expression to approximate $E(Y_I)$, assuming that the number of packets at the beginning of an idle period equals n_1. This leads to an approximation of n_0 as a nonlinear function of n_1. This function, combined with the simple relation that $n_0 = n_1 - p\mu_B$, will be used to find n_1.

We first derive an approximation to $E(Y_I)$ as follows:

$$E(Y_I) \approx \int_0^{\infty} E(Y_v | Y_0 = n_1) f_I(v) dv. \tag{7}$$

To evaluate (7), we break the integral into three parts. Let $m(v, n_1) = E(Y_v | Y_0 = n_1)$. We have

$$E(Y_I) \approx \int_0^{\frac{n_1}{1-p}} f_I(v) m(v, n_1) dv + \int_{\frac{n_1}{1-p}}^{\Gamma} f_I(v) m(v, n_1) dv + \int_{\Gamma}^{\infty} f_I(v) m(v, n_1) dv. \tag{8}$$

Here we have used the fact that given $Y_0 = n_1$, the average number of time slots required for the number of SU packets to first reach zero is $\frac{n_1}{1-p}$. Hence, to ensure system stability, the threshold Γ must satisfy $\Gamma > \frac{n_1}{1-p}$. Next, we compute $m(v, n_1)$ for each integral term in (8).

First, when $v < \frac{n_1}{1-p} < \Gamma$, we have $m(v, n_1) \approx n_1 + v(p-1)$ because during an idle period of length v, on average there are a total of vp new packet arrivals and a maximum of v packet departures. Second, when $\frac{n_1}{1-p} < v < \Gamma$, we have $m(v, n_1) \approx p$. This is because on average the

SU queue length reaches zero after $\frac{n_1}{1-p}$ time slots. Each new packet that arrives afterwards will be transmitted during the next time slot, except for the new arrival that occurs in the last slot of the idle period. This implies that with probability p there is one packet at the end of the idle period and with probability $1-p$ there is no packet in the queue. Third, when $\frac{n_1}{1-p} < \Gamma < v$, the SU packets that are still in the system by the end of the idle period are new arrivals from $[\Gamma, v]$. Hence, we have $m(v, n_1) \approx p \cdot (v - \Gamma)$.

When the system is in steady-state, we have $E(Y_I) = n_0 = n_1 - p\mu_B$. It follows from (8) that

$$n_1 - p\,\mu_B \approx \int_0^{\frac{n_1}{1-p}} (n_1 + v(p-1)) f_I(v)\, dv + \int_{\frac{n_1}{1-p}}^{\Gamma} p f_I(v)\, dv + \int_{\Gamma}^{\infty} p \cdot (v - \Gamma) f_I(v)\, dv. \quad (9)$$

We can solve (9) to obtain n_1, sometimes in closed-form, as shown in the examples in Section V-D.

B. Delay Approximation

We first introduce some notations. Let $d_I(x, v)$ and $d_B(x, u)$ denote the average delay of a packet that arrives x unit times after the start of an idle period of length v, or after the start of a busy period of length u, respectively. The total average delay due to packets that arrive during the idle period of a cycle, and the busy period of a cycle, are then given by $W_I = \int_0^\infty \int_0^v d_I(x, v) dx f_I(v) dv$ and $W_B = \int_0^\infty \int_0^u d_B(x, u) dx f_B(u) du$, respectively. We then apply the renewal theory [10] to obtain an expression for the average delay of a packet W as

$$W = \frac{1}{\mu_B + \mu_I}(W_I + W_B), \quad (10)$$

where the term $\mu_B + \mu_I$ is the average length of a busy/idle cycle.

1) Approximation of W_I: The average delay of a packet depends on the arrival time x, as well as other parameters including Γ, v, n_1. We find that the following three classes of packets contribute dominantly to W_I. The average delay of each class of packet is computed as follows.

A packet is defined as a class 1 packet if $v > \Gamma$ and $x \in (\Gamma, v)$. Such a packet will remain in the queue by the end of this idle period, and hence will experience a substantial delay. We approximate $d_I(x, v)$ by $d_I(x, v) \approx (v - x) + p \cdot (x - \Gamma) + \mu_B$, where $v - x$ is the residual idle time, $p \cdot (x - \Gamma)$ is the queueing delay due to packet arrivals in $[\Gamma, v]$, and μ_B is the average delay incurred by the next busy period. Here we have ignored any remaining SU packets in the queue by time Γ, and we also assume that this class 1 packet will be transmitted during the next busy-idle cycle. Hence, the total average delay due to class 1 SU packets is given by

$$L_1 = \int_{\Gamma}^{\infty} \Big(\int_{\Gamma}^{v} d_I(x,v)\ dx \Big) f_I(v) dv \approx \int_{\Gamma}^{\infty} (v-\Gamma) \Big(\mu_B + \frac{1+p}{2}(v-\Gamma) \Big) f_I(v) dv. \tag{11}$$

A packet is defined as a class 2 packet if $v > \frac{n_1}{1-p}$ and $x \in (0, \frac{n_1}{1-p})$. Such a packet is likely to be transmitted before the end of the idle period, and thus $d_I(x,v) \approx n_1 + px - x = n_1 + (p-1)x$. Hence, we have

$$\begin{aligned} L_2 &= \int_{\frac{n_1}{1-p}}^{\infty} \Big(\int_{0}^{\frac{n_1}{1-p}} d_I(x,v)\ dx \Big) f_I(v) dv \approx \int_{\frac{n_1}{1-p}}^{\infty} \Big(\int_{0}^{\frac{n_1}{1-p}} (n_1 + (p-1)x)\ dx \Big) f_I(v) dv \\ &= \frac{n_1^2}{2(1-p)} \int_{\frac{n_1}{1-p}}^{\infty} f_I(v) dv. \end{aligned} \tag{12}$$

A packet is defined as a class 3 packet if $v \in (0, \frac{n_1}{1-p})$. Such a packet will not be transmitted until the next busy-idle cycle, which leads to $d_I(x,v) \approx n_1 + (p-1)x + \mu_B$. Thus,

$$\begin{aligned} L_3 &= \int_{0}^{\frac{n_1}{1-p}} \Big(\int_{0}^{v} d_I(x,v)\ dx \Big) f_I(v) dv \approx \int_{0}^{\frac{n_1}{1-p}} \Big(\int_{0}^{v} \Big[n_1 + \mu_B + (p-1)x \Big) dx \Big] f_I(v) dv \\ &= \int_{0}^{\frac{n_1}{1-p}} \Big[(n_1 + \mu_B) v + (p-1) \frac{v^2}{2} \Big] f_I(v) dv. \end{aligned} \tag{13}$$

We then combine (11), (12), and (13) to obtain $W_I \approx L_1 + L_2 + L_3$.

2) Approximation of W_B: Consider a packet that arrives x unit times after the start of a busy period. Then $d_B(x,u)$ includes only the average queueing delay $n_0 + px$ and the residual busy time $u - x$, assuming that the packet is transmitted in the current busy/idle cycle. If a packet is not transmitted until the next busy-idle cycle, for instance, if $n_0 + px > \min(v, \Gamma)$, where v is the length of the idle period following the current busy period, then $d_B(x,u)$ has to include additional delay due to the residual idle time beyond Γ, and the delay due to the next busy period. Thus, we write $W_B \approx R_B + R_N$, where

$$R_B = \int_{0}^{\infty} \int_{0}^{u} (n_0 + px + u - x)\ dx f_B(u) du = n_0 \mu_B + \frac{1+p}{2} \int_{0}^{\infty} u^2 f_B(u) du, \tag{14}$$

and let R_N include the additional delay associated with the packets that remain in the queue by the end of the current busy-idle cycle. These will wait for another busy-idle cycle before transmission. Next, we classify all SU packets that contribute to R_N into three classes and let R_{N_i} denote the contribution of class i packet to R_N, where $i = 1, 2, 3$.

A packet is defined as a class 1 packet if $v < n_0 < \Gamma$. Such a packet has to wait for one more busy period for transmission, and thus

$$R_{N_1} \approx \int_0^{n_0} \Big(\int_0^{\infty} \Big(\int_0^{u} \mu_B dx \Big) f_B(u) du \Big) f_I(v) dv = \mu_B^2 \int_0^{n_0} f_I(v) dv. \tag{15}$$

A packet is defined as a class 2 packet if $v \in (n_0, \Gamma)$ and $x \in (\frac{v-n_0}{p}, u)$. Such a packet will also have to wait for one more busy period for transmission.

$$R_{N_2} \approx \int_{n_0}^{\Gamma} \Big(\int_{\frac{v-n_0}{p}}^{\infty} \Big(\int_{\frac{v-n_0}{p}}^{u} \mu_B dx \Big) f_B(u) du \Big) f_I(v) dv. \tag{16}$$

A packet is defined as a class 3 packet if $v \in (\Gamma, \infty)$ and $x \in (\frac{\Gamma-n_0}{p}, u)$. Such a packet will have to wait for the residual idle time $v - \Gamma$ and one busy period for transmission. Hence, we obtain

$$R_{N_3} \approx \int_{\Gamma}^{\infty} \Big(\int_{\frac{\Gamma-n_0}{p}}^{\infty} \Big(\int_{\frac{\Gamma-n_0}{p}}^{u} (\mu_B + v - \Gamma) dx \Big) f_B(u) du \Big) f_I(v) dv. \tag{17}$$

Using (14)–(17), we obtain $W_B \approx R_B + R_{N1} + R_{N2} + R_{N3}$.

Finally, we put things together to obtain the following approximation for W as

$$W \approx \frac{1}{\mu_B + \mu_I} \Big(L_1 + L_2 + L_3 + R_B + R_{N1} + R_{N2} + R_{N3} \Big). \tag{18}$$

C. Collision Approximation

Assuming that on average there are a total of n_1 packets at the beginning of an idle cycle. It then takes approximately $\frac{n_1}{1-p}$ time slots for the SU queue length to reach zero. If the PU returns before $\frac{n_1}{1-p}$, a collision will occur with probability 1. If the PU returns between $[\frac{n_1}{1-p}, \Gamma]$, a collision occurs with probability p, because this is the probability that there is one SU packet in transmission during the time slot that the PU returns. This yields the following approximation

$$p_c \approx \frac{1}{E(N_p)} \Big(\int_0^{\frac{n_1}{1-p}} f_I(v) dv + p \int_{\frac{n_1}{1-p}}^{\Gamma} f_I(v) dv \Big). \tag{19}$$

D. Examples

The approximations for W and p_c given in (18) and (19) are applicable to general busy and idle distributions. Next, we present several examples in which we obtain closed-form expressions of such approximations.

Example 1: Fixed Busy Distribution and Uniform Idle Distribution. Assume that $f_I(t)$ follows a uniform distribution in $[0, 2\mu_I]$, and the busy period is fixed to be μ_B. We first solve (9) to obtain

$$n_1 \approx 2(1-p)\mu_I - \sqrt{4(1-p)^2\mu_I^2 - p(1-p)\Big[\big(2\mu_I - \Gamma\big)^2 + 4\mu_B\mu_I\Big]}. \tag{20}$$

For the delay approximation (18), we apply (11)–(13) to obtain

$$\begin{aligned} &L_1 \approx \frac{1}{12\mu_I}(2\mu_I - \Gamma)^3(1+p) + \frac{\mu_B}{4\mu_I}(2\mu_I - \Gamma)^2, \;\; L_2 \approx \frac{n_1^2}{4(1-p)\mu_I}\Big(2\mu_I - \frac{n_1}{1-p}\Big), \\ &L_3 \approx \frac{n_1^2(n_1+\mu_B)}{4\mu_I(1-p)^2} - \frac{n_1^3}{12\mu_I(1-p)^2}, \end{aligned} \tag{21}$$

and apply (14)–(17) to obtain

$$R_B \approx n_0\mu_B + (1+p)\frac{\mu_B^2}{2}; \quad R_{N_1} \approx \frac{n_0\mu_B^2}{2\mu_I}; \quad R_{N_2} \approx \frac{\mu_B^3 p}{4\mu_I}; \quad R_{N_3} = 0. \tag{22}$$

The collision probability can be computed from (19) as

$$p_c \approx \frac{1}{2\mu_I E(N_p)}(n_1 + p\Gamma). \tag{23}$$

Example 2: Exponential Busy distribution and Uniform Idle Distribution. Since this example differs from Example 1 only in the busy distribution, both (21) and (23) remain unchanged, and we replace (22) by

$$\begin{aligned} &R_B \approx \mu_B^2(1+p) + n_0\mu_B; \quad R_{N_1} \approx \frac{n_0\mu_B^2}{2\mu_I}; \quad R_{N_2} \approx \frac{p\mu_B^3}{2\mu_I}\Big(1 - e^{\frac{n_0-\Gamma}{p\,\mu_B}}\Big) \\ &R_{N_3} \approx \mu_B(2\mu_I - \Gamma)\Big(\frac{1}{2} - \frac{\Gamma}{4\mu_I} + \frac{\mu_B}{2\mu_I}\Big)e^{\frac{n_0-\Gamma}{p\mu_B}}. \end{aligned} \tag{24}$$

Example 3: Fixed Busy Distribution and Weibull Idle Distribution. We consider a Weibull idle distribution with scale parameter λ and shape parameter 2 such that $f_I(t) = \frac{2t}{\lambda^2}e^{-\frac{t^2}{\lambda^2}}$. To estimate

n_1, we solve (9) to obtain

$$n_1 \approx \lambda(1-p)\ \text{erfinv}\Big(\frac{p}{1-p}\text{erfc}(\frac{\Gamma}{\lambda}) + \frac{2p\ \mu_B}{(1-p)\sqrt{\pi}\lambda}\Big). \tag{25}$$

Here erf(x) is the error function, erfinv(x) is the inverse error function, and erfc(x) is the complementary error function. For the delay approximation (18), we apply (11)–(13) to obtain

$$\begin{aligned} L_1 &\approx \mu_I \frac{\lambda e^{-\frac{\Gamma^2}{\lambda^2}} - \Gamma\sqrt{\pi}\text{erfc}(\frac{\Gamma}{\lambda})}{2\Gamma(1.5)} + \frac{\mu_B\mu_I}{\Gamma(1.5)}\frac{\sqrt{\pi}}{2}\text{erfc}(\frac{\Gamma}{\lambda}), \quad L_2 \approx \frac{n_1^2}{2(1-p)}e^{-\frac{n_1^2}{\lambda^2(1-p)^2}} \\ L_3 &\approx \mu_I(n_1+\mu_B)\Big(\lambda\frac{\sqrt{\pi}}{2}\text{erf}(\frac{n_1}{\lambda(1-p)}) - \frac{n_1}{1-p}e^{-\frac{n_1^2}{\lambda^2(1-p)^2}}\Big) \\ &\quad - \mu_I\frac{(1-p)}{2}\lambda^2\Big[1-\big(1+\frac{n_1^2}{\lambda^2(1-p)^2}\big)e^{-\frac{n_1^2}{\lambda^2(1-p)^2}}\Big]. \end{aligned} \tag{26}$$

and apply (14)–(17) to obtain

$$\begin{aligned} R_B &\approx \frac{\mu_B^2}{2}(1+p) + n_0\mu_B; \quad R_{N_1} \approx \mu_B^2(1-e^{-\frac{n_0^2}{\lambda^2}}), \quad R_{N_3} = 0. \\ R_{N_2} &\approx \mu_B\Big(-\mu_B - \frac{n_0}{p} + p^2\mu_B + pn_0\Big)e^{-\frac{(p\mu_B+n_0)^2}{\lambda^2}} + \\ &\quad \mu_B(\frac{n_0}{p} - pn_0 + \mu_B)e^{-\frac{n_0^2}{\lambda^2}} + \mu_B p\lambda\frac{\sqrt{\pi}}{2}\Big(\text{erf}(\frac{n_0}{\lambda}) - \text{erf}(\frac{p\mu_B+n_0}{\lambda})\Big), \end{aligned} \tag{27}$$

The collision probability is computed from (19)

$$p_c \approx \frac{1}{E(N_p)}\Big(e^{-\frac{n_1^2}{\lambda^2(1-p)^2}}(p-1) + e^{-\frac{1}{\lambda^2}} - pe^{-\frac{\Gamma^2}{\lambda^2}}\Big). \tag{28}$$

VI. Numerical Results

In this section, we present numerical results to examine the performance of the proposed threshold policy and the accuracy of the developed analysis. From Section VI-A to Section VI-D, we assume perfect sensing with zero sensing time and error-free sensing outcome. The PU/SU packets have the same fixed length. In Section VI-E we consider the imperfect sensing scenarios.

A. Comparison of Threshold Policy and MDP Policy

In Fig. 2, we compare the delay performance of the threshold policy and the MDP policy computed from the discounted MDP formulation described in [9]. Here we assume that the busy period is fixed to be $\mu_B = 100$ and each PU/SU packet has a fixed packet length of one. The length of the idle period is uniformly distributed in $[0, 300]$ with mean $\mu_I = 150$. The capacity of this system equals $C = 0.114$ under the collision probability constraint $\eta = 0.001$ [6]. For each SU packet arrival rate p, we can find the corresponding threshold Γ for the threshold policy such that $p_c = \eta$ and then find the delay W under this threshold policy. Using the MDP formulation, for each p, we adjust the cost C_m to find an MDP policy so that $p_c = \eta$ and evaluate its delay. In Fig. 2, we plot the delay achieved by the threshold policy and the MDP policy as a function of p. It is shown that the threshold policy performs very closely to the MDP policy for the entire range of p considered. In Fig. 3, we compare the two-dimensional transmission regions of the MDP policy and the threshold policy for $p = 0.11$. The threshold policy has $\Gamma = 94$. The MDP policy is a function of both the elapsed idle time t and the queue length M. The SU transmits when (t, M) falls into the region to the left of line 2 shown in Fig. 3. As opposed to the MDP policy, the threshold policy is independent of the queue length and its transmission region is to the left of line 1 (corresponding to $t < \Gamma = 94$). It is interesting to note that the corner point of the MDP curve $(t, M) = (91, 1)$ is very close to the threshold Γ of the threshold policy. While the transmission region of the MDP policy is larger than that of the threshold policy, the two policies yield similar delay performance. This is because the probability that (t, M) belongs to the middle region (between line 1 and line 2) is small. The effect on the delay performance is thus negligible.

B. Accuracy of Markov Chain Analysis and Analytical Approximations

In Fig. 4 and Fig. 5, we examine the accuracy of the Markov chain analysis, developed in Section IV, and the analytical approximations for the delay and the collision probability, developed in Section V, for the threshold policy under various combinations of busy and idle distributions. Here we fix the SU packet arrival rate and plot W and p_c as functions of the threshold Γ. The curves for the Markov chain analysis are numerically evaluated from (5) for W and (6) for p_c. The analytical approximations for W and p_c are obtained from (18) and (19), using the closed-form expressions presented in Section V-D.

In Fig. 4, we consider an exponential busy distribution and a uniform idle distribution, assuming $p = 0.11$. It shows that the Markov chain analysis matches the simulation results perfectly. For each simulation point, we run a total of 10^7 busy-idle cycles to achieve reliable results. For the Markov chain analysis, we find that a total of 200 states is sufficient to obtain accurate numerical results. Following Example 2 in Section V-D, we evaluate the closed-form analytical approximations for W and p_c. We observe that both approximations are very tight, yielding errors less than $3\% - 4\%$. While similar results have been obtained for Example 1 under a fixed busy distribution and a uniform idle distribution, the results are not included here due to space limitation.

In Fig. 5, we consider the fixed busy distribution and the Weibull idle distribution. Since the Weibull idle distribution yields a higher capacity of $C = 0.213$, we choose a higher $p = 0.2$ to operate near capacity. While the Markov chain analysis remains accurate, the analytical approximations from Example 3 in Section V-D become looser compared to the case of the uniform idle distribution shown in Fig. 4. The error of the delay approximation is roughly 5% except for the left-most point in Fig. 5 (a), corresponding to $\Gamma = 66$. Note that the delay increases rapidly in this region, and some of the SU packets might need to wait for more than two busy-idle cycles for transmission. This is not taken into account in the analysis presented in Section V, which may contribute to the inaccuracy of the approximation at this point. The error of the collision approximation is about 8% in the case.

C. Comparisons of Various Busy Distributions

In Fig. 6, we examine the effect of the busy distribution on the delay and collision probability. We consider four different busy distributions with the same mean: the exponential distribution, the uniform distribution, the Weibull distribution, and the fixed busy distribution. It is shown in Fig. 6 (a) that the busy distribution affects the delay significantly. Out of these four distributions, the exponential busy distribution and the fixed busy distribution induce the largest delay and the smallest delay, respectively. On the other hand, given the same threshold, Fig. 6 (b) shows that the collision probability changes only slightly with busy distributions.

D. Comparisons of Various Idle Distributions

Next, we examine the effect of idle distribution on the SU delay under the packet collision constraint. We assume a uniform busy distribution and consider three idle distributions with the same mean $\mu_I = 150$: uniform, Weibull, and exponential. For the first two idle distributions and for each p, we determine Γ of the threshold policy such that the collision probability $p_c = \eta = 0.001$. For the exponential idle distribution, due to its memoryless property, it is optimal to use a greedy policy, under which the SU transmits whenever the channel is sensed idle and the SU queue is nonempty. Given $\eta = 0.001$, the time capacity $C = 0.06,\ 0.114,\ 0.213$, for the exponential, uniform, and Weibull distribution, respectively.

In Fig. 7, we plot the delay achieved by the threshold policy as a function of p under the collision constraint of $\eta = 0.001$. For each idle distribution, there exists some p^* such that when $p < p^*$, the threshold policy becomes the greedy policy. We find that $p^* = 0.06,\ 0.075,\ 0.095$ for the exponential, uniform, and Weibull distribution, respectively. Fig. 7 shows that the delay achieved by the threshold policy under these three idle distributions are similar in the region $p < p^*$ where the greedy policy is the optimal policy. For the exponential distribution, $p^* = 0.06$ is the highest arrival rate such that p_c does not exceed η. In comparison, the capacity of the uniform distribution is higher. Therefore, in the region $p \in [0.075, 0.114)$, a threshold policy can be found to ensure that $p_c = \eta$ at the cost of increased delay. We note that the delay achieved by the threshold policy increases rapidly as p approaches the capacity of 0.114. The Weibull distribution has the highest capacity. When $p \in [0.095, 0.2]$, the delay achieved by the threshold policy increases with p, but at a slower rate than that of the uniform idle distribution.

E. Imperfect Sensing

The analysis developed in this paper assumes perfect sensing where the sensing time is zero and the sensing outcome is error-free. Next, we examine more realistic scenarios in which the sensing is imperfect, and compare the delay and collision probabilities with those of perfect sensing. In Fig. 8, we plot W and p_c for various sensing scenarios. The busy distribution is a uniform distribution with $\mu_B = 100$ and the idle distribution is a uniform distribution with $\mu_I = 150$. The PU packet has a fixed length of 1ms. Each SU slot is 1ms long, in which 5% is for sensing, and 95% is for transmission. We first consider three imperfect sensing scenarios where the miss detection probability $\gamma_{\text{md}} = 10^{-3}$, and the false alarm probability γ_{f} varies from 0.01 to 0.2. Note that it is important to set γ_{md} low in order to limit collision with the PU. Due to the possibility of false alarm, the SU will determine the transition from an idle period to a busy period only when, within an idle period, the PU channel is detected to be busy for several consecutive sensing periods. Similarly, due to the possibility of miss detection, the transition from a busy period to an idle period is determined only when, within a busy period, the PU channel is detected to be idle for a few consecutive sensing periods. As shown in Fig. 8, both W and p_c decrease as γ_{f} decreases. As a performance benchmark, we also consider a fourth imperfect sensing scenario, for which the system setup, such as the SU slot duration, and the lengths of the sensing period and the transmission period, is identical to the other three imperfect sensing scenario except that now we set $\gamma_{\mathrm{md}} = \gamma_{\mathrm{f}} = 0$. This is the best performance that one can achieve for the given system setup with nonzero sensing time. Fig. 8 shows that indeed the curves for the fourth imperfect sensing scenario with $\gamma_{\mathrm{md}} = \gamma_{\mathrm{f}} = 0$ are very close to that of the analytical curve obtained from (18) and (19), which assumes error-free sensing outcome and zero sensing time. We also observe from Fig. 8 (b) that for p_c, the analytical curve gives close approximation to all four curves of imperfect sensing scenarios, where the largest gap is less than 6%. In comparison, imperfect sensing has a stronger effect on W. As shown in Fig. 8 (a), the gap between the imperfect sensing curves and the analytical curve becomes more pronounced as γ_{f} increases.

VII. Conclusions

In this work, we propose and analyze threshold-based transmission policies to minimize the delay performance of the SU subject to a collision constraint on the PU. Such threshold policies are shown to perform closely to an optimal policy found through a discounted MDP formulation. A novel Markovian approach is developed to analyze the delay and collision probabilities of the threshold policies. This approach treats each busy-idle PU cycle as a one-step transition in the Markov chain, which effectively reduces the state-space of the Markov chain to facilitate numerical computations. Furthermore, we develop analytical expressions to approximate the delay and collision performance of the threshold policies under general busy and idle distributions. The accuracy of the proposed approximation is confirmed numerically for several commonly used busy and idle distributions. Furthermore, we show that the busy time distribution significantly impacts the delay performance of the SU, while the collision probability and the threshold policy are largely determined by the idle distribution. This is a dual observation of the results of [6], [5], which shows that the PU idle time distribution largely determines the SU capacity. Future work includes extension of the existing analysis to more general scenarios such as arbitrary SU arrival processes, multiple PU channels, and multiple SUs.

Appendix A
Computation of the transition probability matrix

During the busy period we have $X_n = X_{n-1} + Q$, where Q is a Bernoulli random variable with parameter p. For the n-th idle time slot such that $n \leq \Gamma$, we have $Y_n = \max(Y_{n-1} - 1 + Q, 0)$ because in this time slot we have at most one SU packet departure and one new packet arrival. Note that once $Y_n = 0$ or $Y_n = 1$ for some time slot n, then we have $Y_v = 0$ or 1 for every $n \leq v \leq \Gamma$. This can be used to show that, for every $j \geq 2$ and $v \leq \Gamma$,

$$P(Y_v = j|X_0 = i, B = u) = \binom{v+u}{v+j-i} p^{v+j-i}(1-p)^{u-j+i}. \tag{29}$$

In (29) we use the fact that since $j \geq 2$, we must have $Y_n \geq 2$ for all $1 \leq n \leq v$ and thus we have exactly v packet departures by time slot v. Hence, the number of new packet arrivals out of u busy slots and v idle slots is $v + j - i$. Then we arrive at (29) using the binomial distribution.

To compute $P(Y_v = j|X_0 = i, B = u)$ for $j = 0, 1$, and $v \leq \Gamma$, we follow the iterative relations:

$$\begin{aligned} P(Y_v = 0|X_0 = i, B = u) &= (1-p)\big[P(Y_{v-1} = 1|X_0 = i, B = u) + P(Y_{v-1} = 0|X_0 = i, B = u)\big], \\ P(Y_v = 1|X_0 = i, B = u) &= (1-p)P(Y_{v-1} = 2|X_0 = i, B = u) + pP(Y_{v-1} = 0|X_0 = i, B = u) \\ &\quad + pP(Y_{v-1} = 1|X_0 = i, B = u) \\ &= (1-p)P(Y_{v-1} = 2|X_0 = i, B = u) + \frac{p}{1-p}P(Y_v = 0|X_0 = i, B = u). \end{aligned}$$

When $v \geq \Gamma + 1$, the SU does not transmit. It is sufficient to take into account the possibility of a new packet arrival to obtain

$$P(Y_{v+1} = j|X_0 = i, B = u) = (1-p)P(Y_v = j|X_0 = i, B = u) + pP(Y_v = j-1|X_0 = i, B = u).$$

APPENDIX B
PROOF OF (5)

We will first compute N using (4). Since $X_n = X_{n-1} + Q$, where Q is a Bernoulli random variable with parameter p, we have $E(X_n) = E(X_0) + np$. Hence,

$$E(X_0 + X_1 + \cdots + X_{B-1}) = \sum_{u=1}^{\infty} P(B=u) \sum_{n=0}^{u-1} E(X_n) = \sum_{u=1}^{\infty} P(B=u) \sum_{n=0}^{u-1} (E(X_0) + np)$$

$$= E(X_0) \sum_{u=1}^{\infty} uP(B=u) + \frac{p}{2}\left(\sum_{u=1}^{\infty} P(B=u)(u-1)u \right) = \mu_B E(X_0) + \frac{p}{2}(\nu_B - \mu_B). \quad (30)$$

To compute $E(Y_0 + Y_1 + \cdots + Y_{I-1})$, we have

$$E(Y_0 + Y_1 + \cdots + Y_{I-1}) = \sum_{v=1}^{\infty} P(I=v) \sum_{n=0}^{v-1} E(Y_n)$$

$$= \sum_{v=1}^{\Gamma} P(I=v) \sum_{n=0}^{v-1} E(Y_n) + \sum_{v=\Gamma+1}^{\infty} P(I=v) \left(\sum_{n=0}^{\Gamma} E(Y_n) + \sum_{n=\Gamma+1}^{v-1} E(Y_n) \right). \quad (31)$$

Note that for every $n \leq \Gamma$, we have $Y_n = \max(Y_{n-1} - 1 + Q, 0)$. It follows that

$$E(Y_n) = E(Y_{n-1}) + (p-1) + P(Y_{n-1} = 0) = E(Y_0) + n(p-1) + \sum_{i=0}^{n-1} P(Y_i = 0).$$

Hence, for every $v \leq \Gamma + 1$, we have

$$\sum_{n=0}^{v-1} E(Y_n) = \sum_{n=0}^{v-1} \left(E(Y_0) + n(p-1) + \sum_{i=0}^{n-1} P(Y_i = 0) \right)$$

$$= vE(Y_0) + (p-1)\frac{(v-1)v}{2} + \sum_{n=0}^{v-1} \sum_{i=0}^{n-1} P(Y_i = 0)$$

$$= vE(Y_0) + (p-1)\frac{(v-1)v}{2} + \sum_{i=0}^{v-2} P(Y_i = 0)(v-1-i). \quad (32)$$

For every $n \geq \Gamma + 1$, we have $Y_n = Y_{n-1} + Q$ and hence $E(Y_n) = E(Y_{n-1}) + p$. It follows that

$$\sum_{n=\Gamma+1}^{v-1} E(Y_n) = \sum_{n=\Gamma+1}^{v-1} \Big(E(Y_\Gamma) + (n-\Gamma)p \Big) = (v-\Gamma-1)E(Y_\Gamma) + \frac{p(v-\Gamma-1)(v-\Gamma)}{2}$$

$$= (v-\Gamma-1)\left(E(Y_0) + (p-1)\Gamma + \sum_{i=0}^{\Gamma-1} P(Y_i = 0) \right) + \frac{p(v-\Gamma-1)(v-\Gamma)}{2}. \quad (33)$$

Noting that $E(Y_0) = p\mu_B + E(X_0)$, we substitute (32) and (33) into (31) and combine the result with (30) and $W{=}N/p - 1$ to obtain (5).

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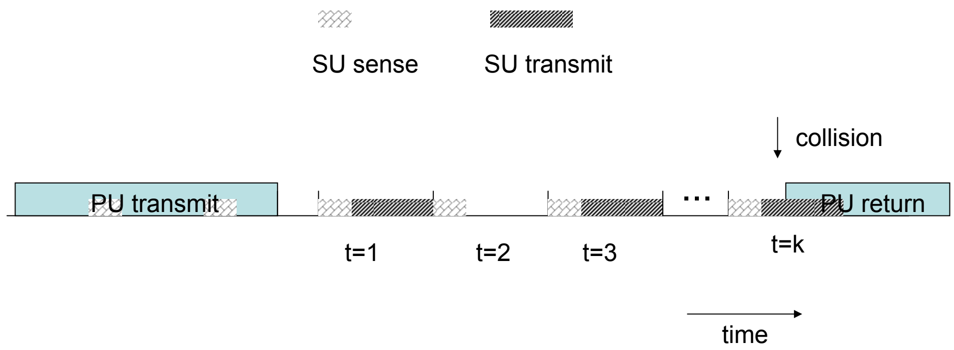


Fig. 1. Illustration of the system model. Each SU slot consists of a sensing period and a transmission period. A collision occurs in the k-th SU slot, when the SU starts transmission after sensing the PU channel to be idle, and the PU returns before the end of the SU transmission period.

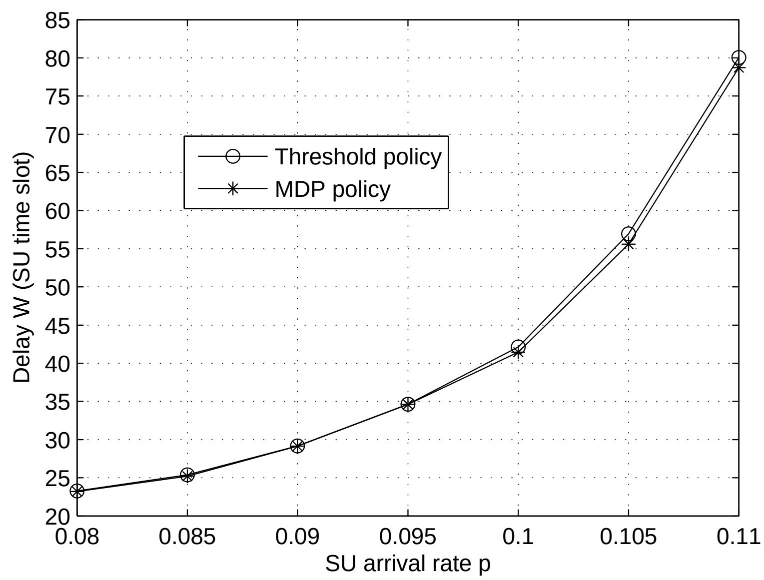


Fig. 2. Comparisons of the threshold policy and the MDP policy for a fixed busy distribution and a uniform idle distribution. For each p, a threshold policy and an MDP policy are found such that $p_c = \eta = 0.001$.

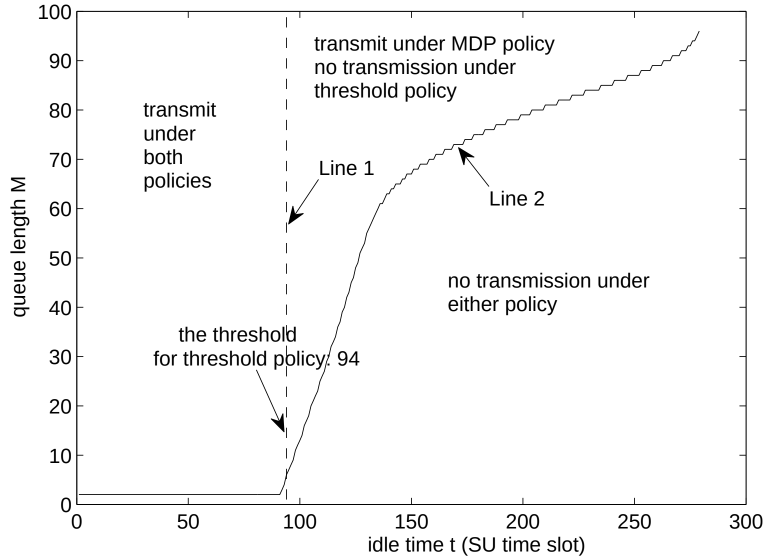
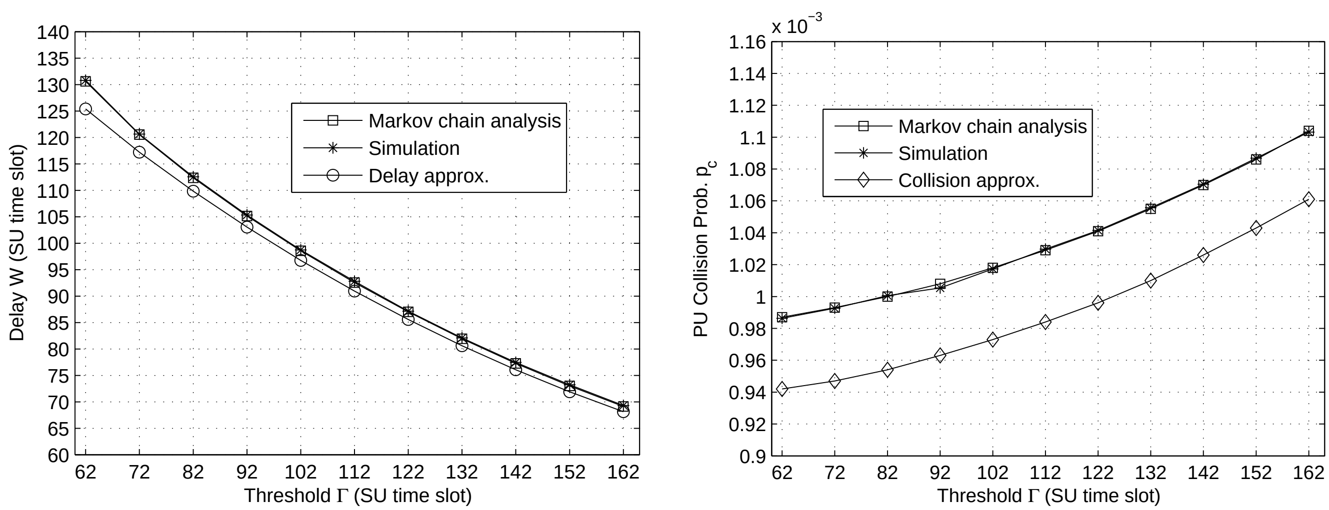
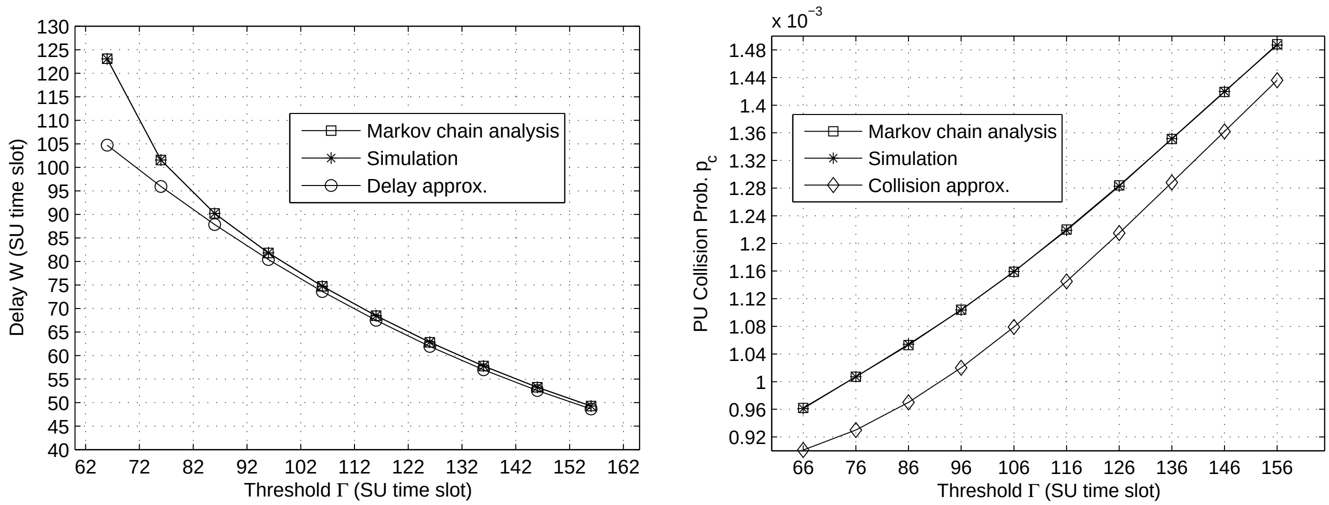


Fig. 3. Transmission regions of the MDP policy and the threshold policy for a fixed busy time distribution and a uniform idle distribution. Assume $p = 0.11$. The two policies are found such that $p_c = \eta = 0.001$.



(a) Delay (b) Collision probability

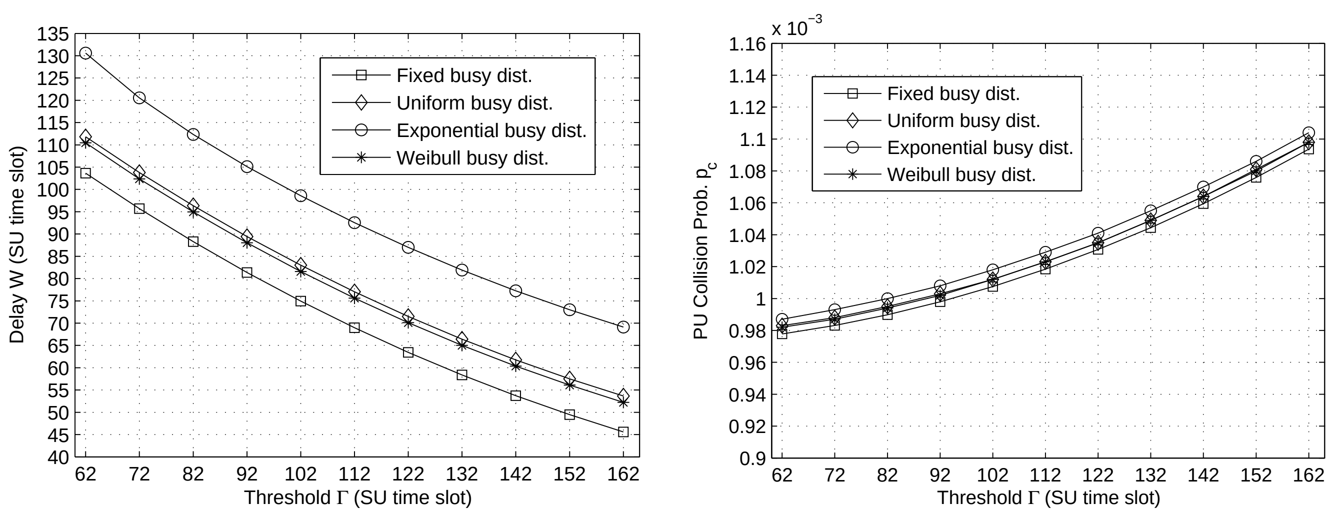
Fig. 4. Performance of threshold policies as a function of Γ. Comparisons of Markov chain analysis (5) and (6), analytical approximations (18) and (19) based on (21), (23), (24), and simulation results for an exponential busy distribution and a uniform idle distribution. Assume $p = 0.11$.



(a) Delay

(b) Collision probability

Fig. 5. Performance of threshold policies as a function of Γ. Comparisons of Markov chain analysis (5) and (6), analytical approximations (18) and (19) based on (26)–(28), and simulation results for a fixed busy time distribution and a Weibull idle distribution. Assume $p = 0.2$.



(a) Delay

(b) Collision probability

Fig. 6. Performance of threshold policies as a function of Γ. Delay and collision probability comparisons of various busy distributions. Assume a uniform idle distribution and $p = 0.11$.

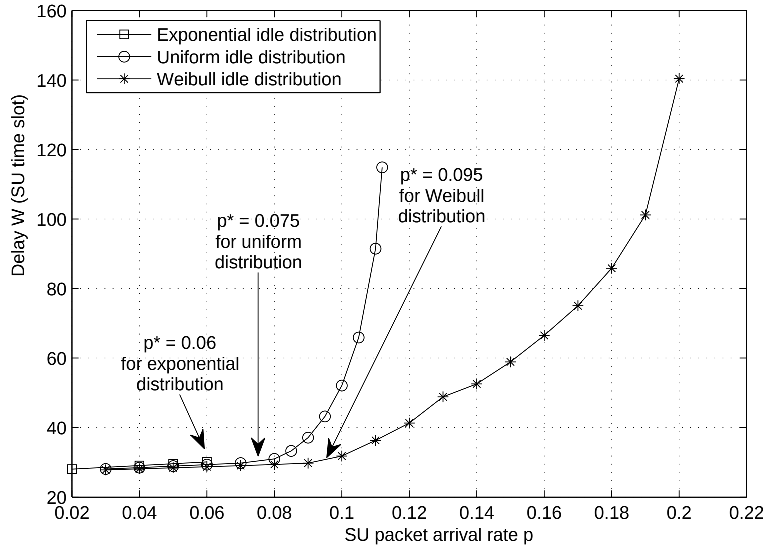
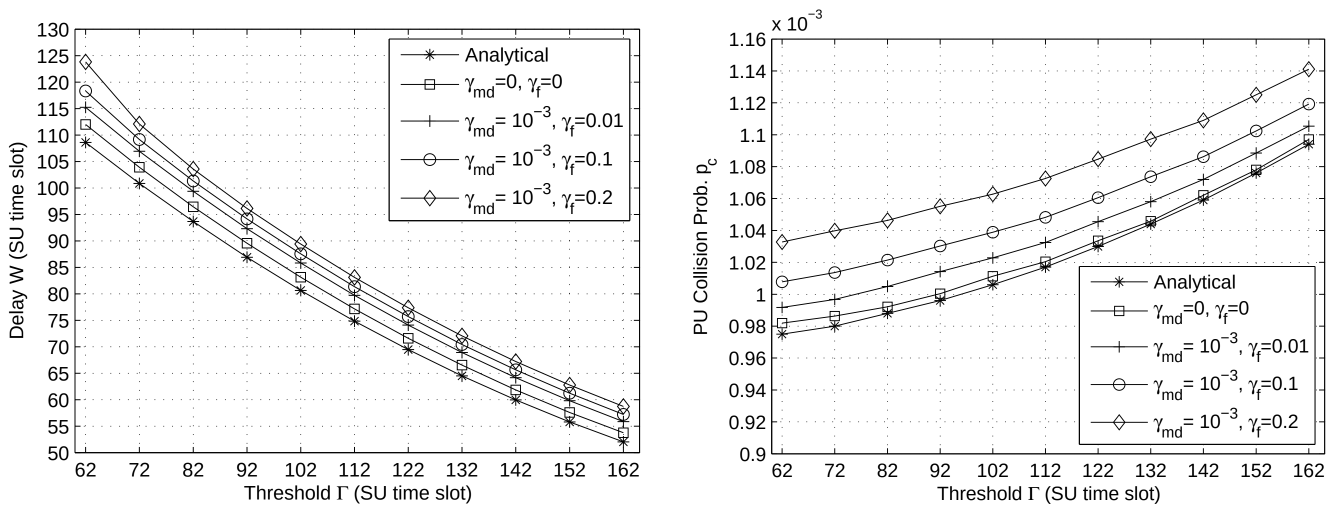


Fig. 7. Delay performance of the threshold policies as a function of p. For each p, a threshold policy with threshold Γ is chosen such that $p_c = \eta = 0.001$. Assume a uniform busy distribution and various idle distributions.



(a) Delay

(b) Collision probability

Fig. 8. Performance of threshold policies as a function of Γ. Comparisons of delay and collision probability for various imperfect sensing scenarios and the analytical approximations from (18) and (19) that assume perfect sensing. Assume a uniform idle distribution and a uniform busy distribution, $p = 0.11$.